# Visualizing the Time Series Behavior of Volatility, Serial Correlation and Investment Horizon

Presentation to:

QWAFAFEW - SF

Presented by:

Ralph Goldsticker, CFA

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# Concerns about standard approaches to modeling risk for investors with longer investment horizons

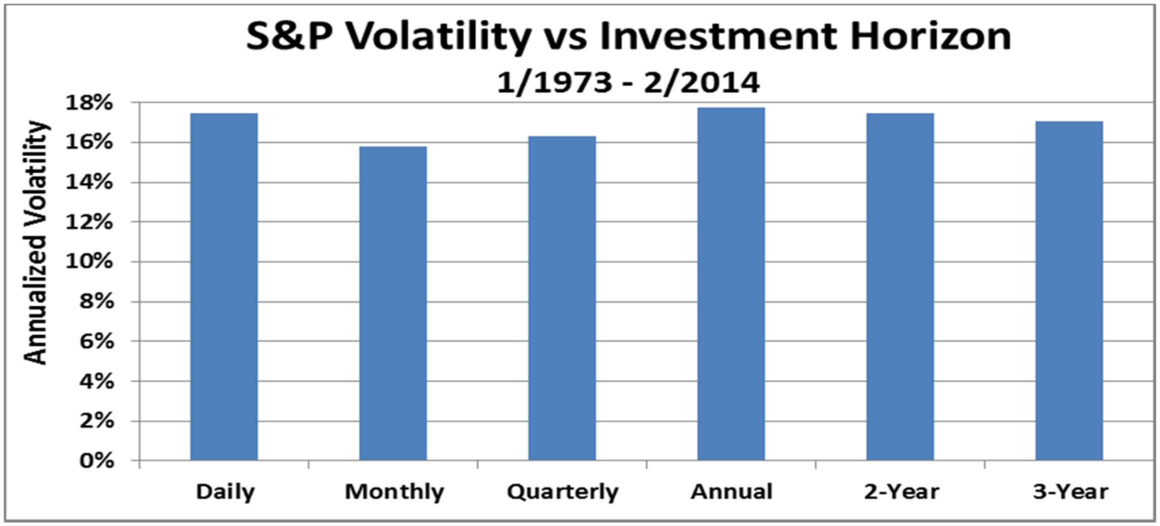
100% quantitative / systematic approaches may not fully capture data and market complexities.

Estimates may be overly dependent on the date sample and sampling frequency.

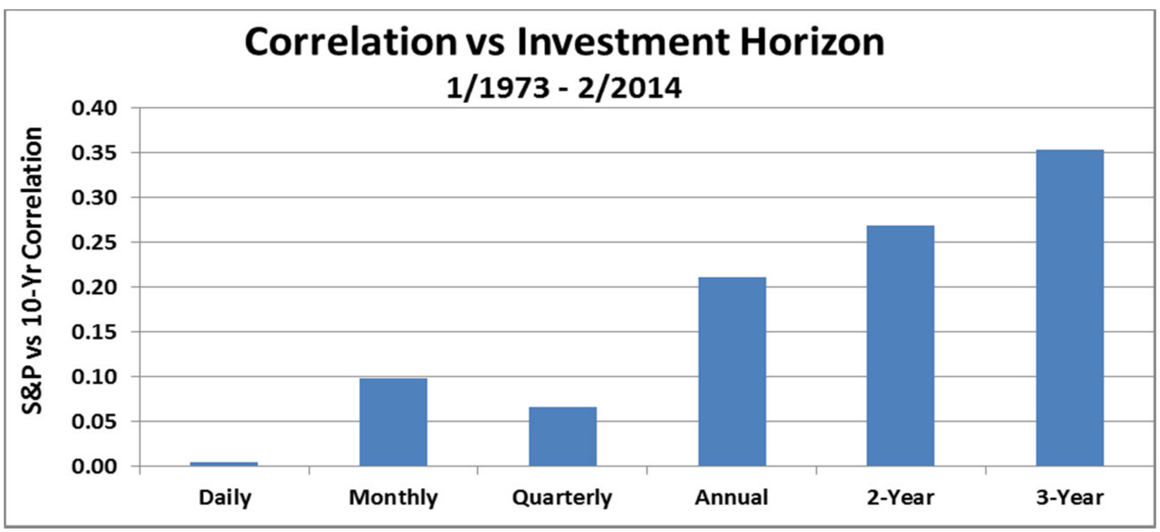
- If volatility and correlations are investment horizon dependent, modeling with shorter-term returns may produce poor estimates of longer-term risk.
- Volatilities, correlations and serial correlations may vary over time.
- Estimates can be sensitive to a few observations.

# Modeling risk using only shorter-term returns may produce poor estimates of longer-term risks.

- Standard deviations may not grow with the square root of time.



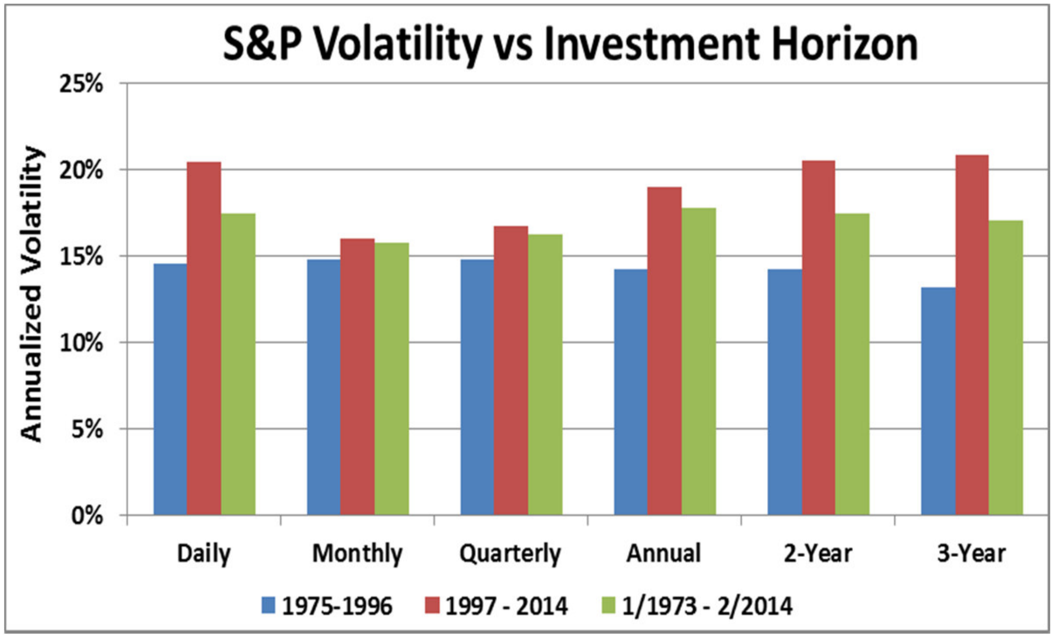

- Correlations may be investment horizon dependent.



# Volatility varied with data window and sampling frequency.

- Volatility is dependent on market and economic environments. Both uncertainty and risk aversion vary through time.
- Time diversification versus momentum depends on the environment.
- Time-based techniques such as exponential weighting and moving window may not properly capture market cycles and regimes.

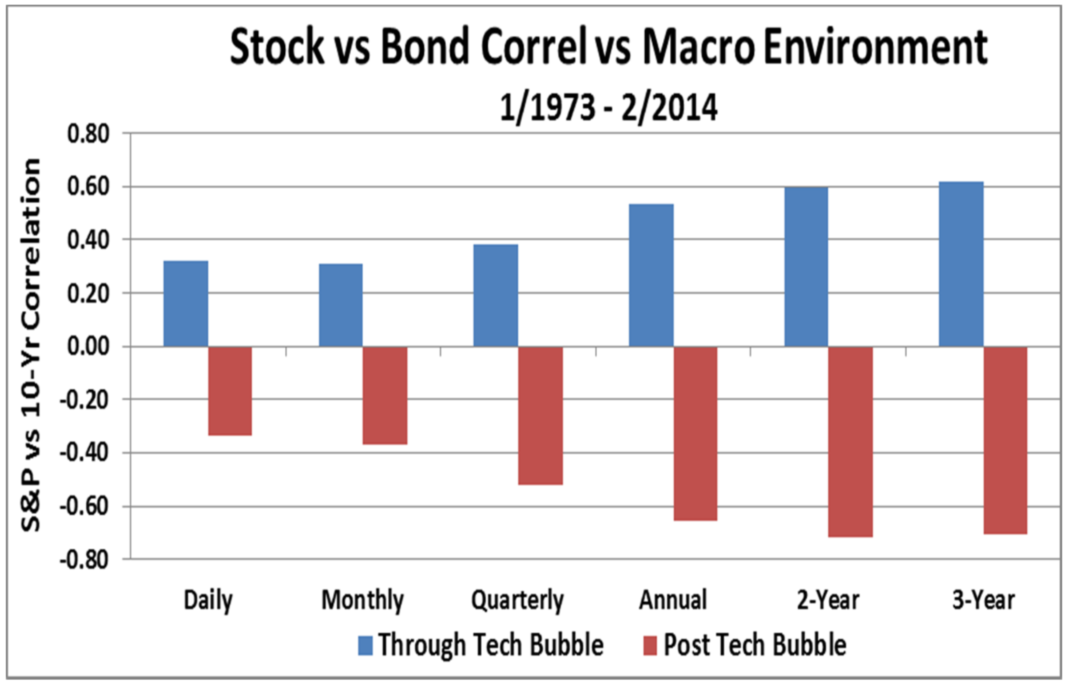
- Volatility was higher after 1996 than before.
- Later period showed short-term reversal and longer-term momentum.
- Early period showed longer-term reversal (time diversification).



# Stock versus bond correlation varied over time. Correlation increased with investment horizon.

- Correlations depend on the underlying environment.
- Time-based techniques such as exponential weighting and moving window may not properly capture market cycles and regimes.

- Stocks and bonds positively correlated through 1999.
- Stocks and bonds negatively correlated since.
- Correlations increased with investment horizon in both periods.



# Objective:
# Modeling risk to match the investment horizon

1. What holding period for returns?
   - Higher frequency returns provide more precise estimates. (More degrees of freedom.)

   But:
   - Volatility may not grow with time.
   - Correlations may depend on holding period.
   - Adjusting for serial correlations and cross-correlations won't capture episodic impacts from different types of news arriving at different frequencies.
     - Cash Flows
     - Growth Rates
     - Discount Rates

2. What data sample do we use?
   - Future more likely to look like recent than distant past.

   But:
   - Distant events provide evidence of what could happen.
   - May have view on which past periods resemble expected future.

# Use judgment to incorporate investment horizon and environment into risk forecasts.

Rather than relying on rules and ever more complex models,

1. Use cumulative contribution charts to visually examine the behavior of volatilities and correlations.
   - Through time
   - As function of return period
2. Use judgment to select the return period and data window that you believe represents the future environment.

# Calculating cumulative contribution to variance

1. Variance $= \dfrac{\sum(\text{Return}_t - \text{Average Return})^2}{\text{Total number of periods} - 1}$

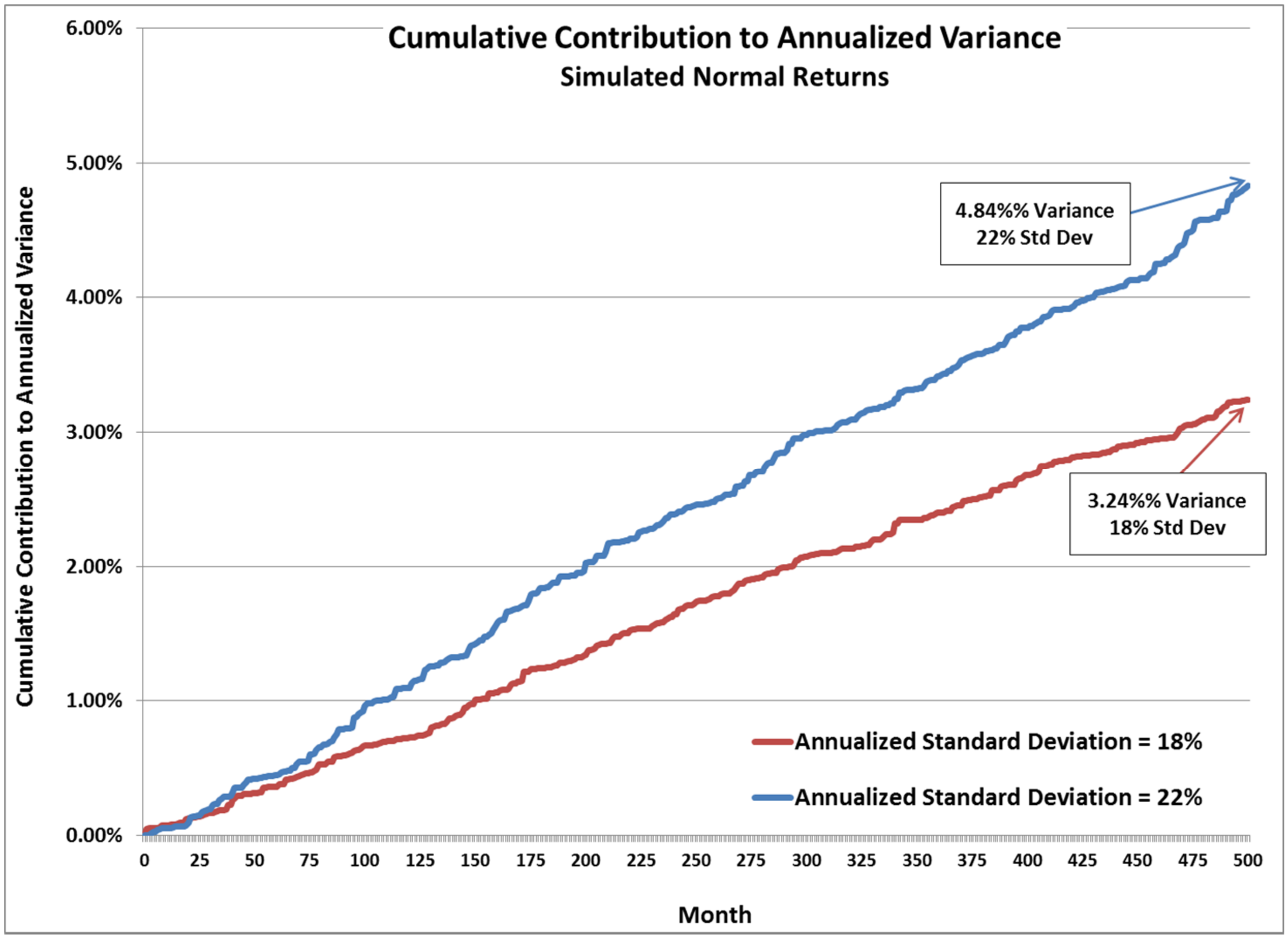
2. Contribution to variance for period t $= \dfrac{(\text{Return}_t - \text{Average Return})^2}{\text{Total number of periods} - 1}$

3. Cumulative contribution to variance $= \dfrac{\sum_1^t(\text{Return}_t - \text{Average Return})^2}{\text{Total number of periods} - 1}$

Note: Use variance rather than standard deviation because variance grows linearly with time.

# Interpreting Cumulative Contribution to Variance Charts
## Constant Volatility (simulated returns)

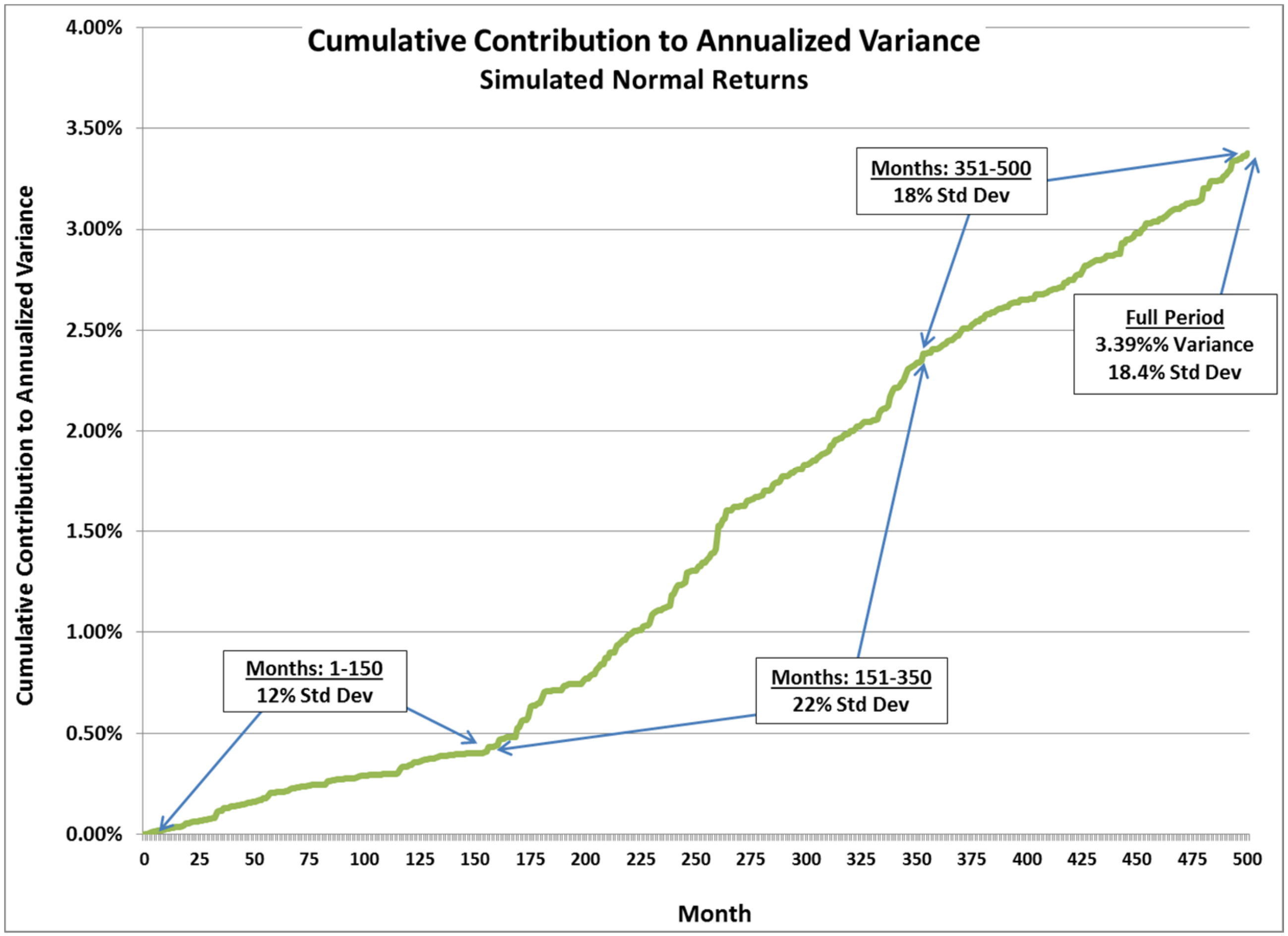
- Higher volatility results in steeper line.
- Lines end at annualized variance.



# Interpreting Cumulative Contribution to Variance Charts

## Time Varying Volatility (simulated returns)

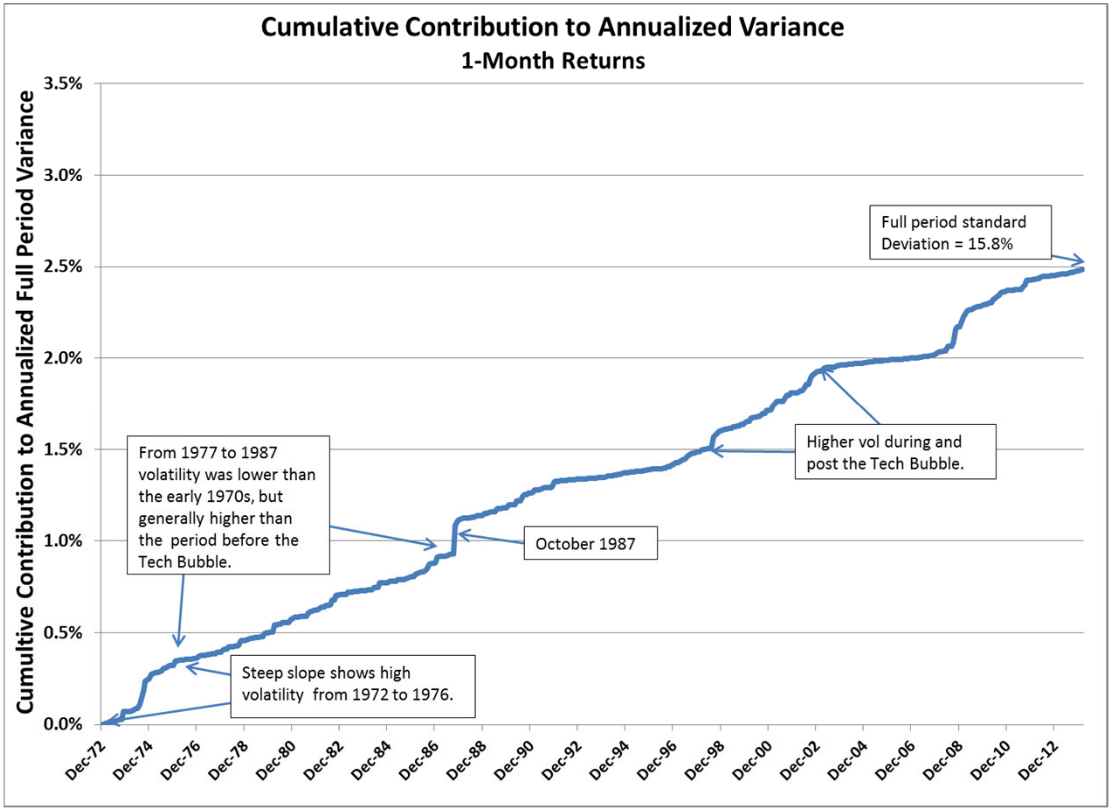
- Slope of line changes as volatility changes.
- Lines end at annualized variance of full sample.



# Examples of cumulative contribution to variance charts

1. Chart of cumulative contribution to 1-month variance
2. Cumulative contribution to variance versus rolling variance.
3. Cumulative contribution to variance versus length of return period
4. Variance estimated using daily versus monthly returns
5. Variance estimated 1-month versus 3- and 12-month returns
6. Variance estimated 12-month versus 24- and 36-month returns

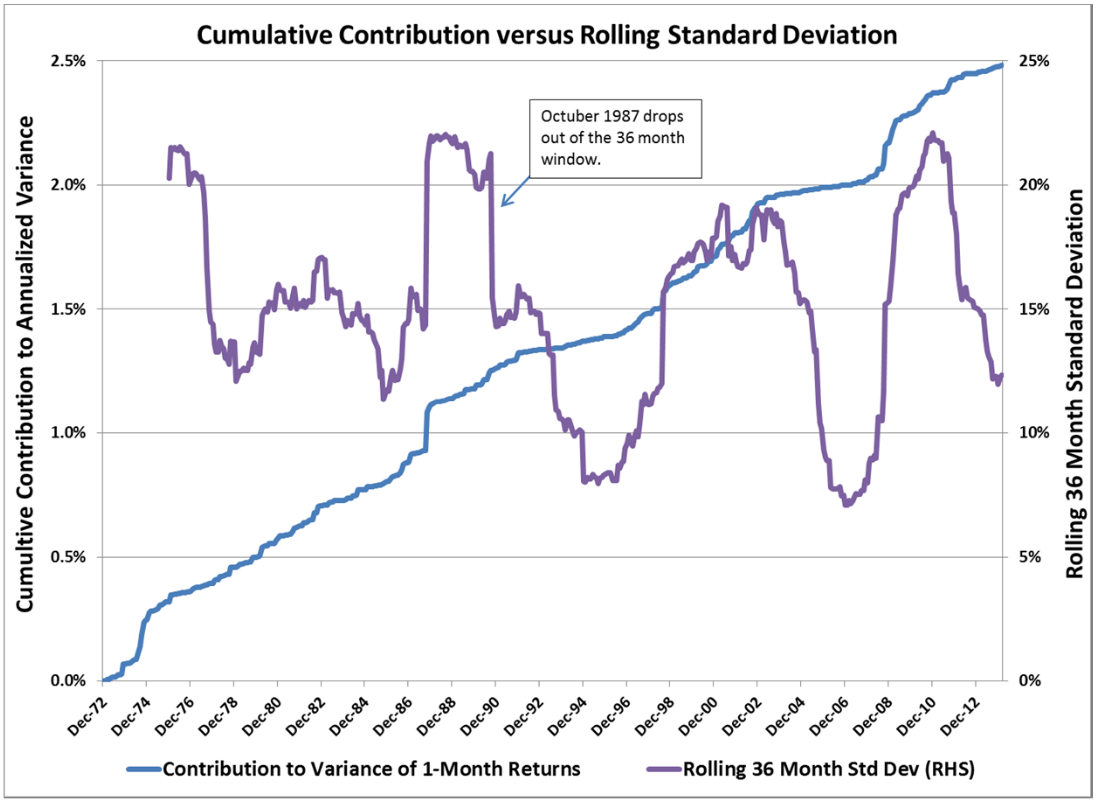
# Cumulative contribution to variance of 1-month returns




Observations:

- Ends at 2.48%% (full period variance)
- Steeper than average slope shows high volatility  from 1972 to 1976.
- Slope of line from 1977 to 1987 shows volatility was lower than early 1970s, but higher than subsequent period.
- Crash of October 1987 is visible.
- Steeper slope shows higher volatility during and post the Tech Bubble.
- *Future will look like the post October 1987 period*

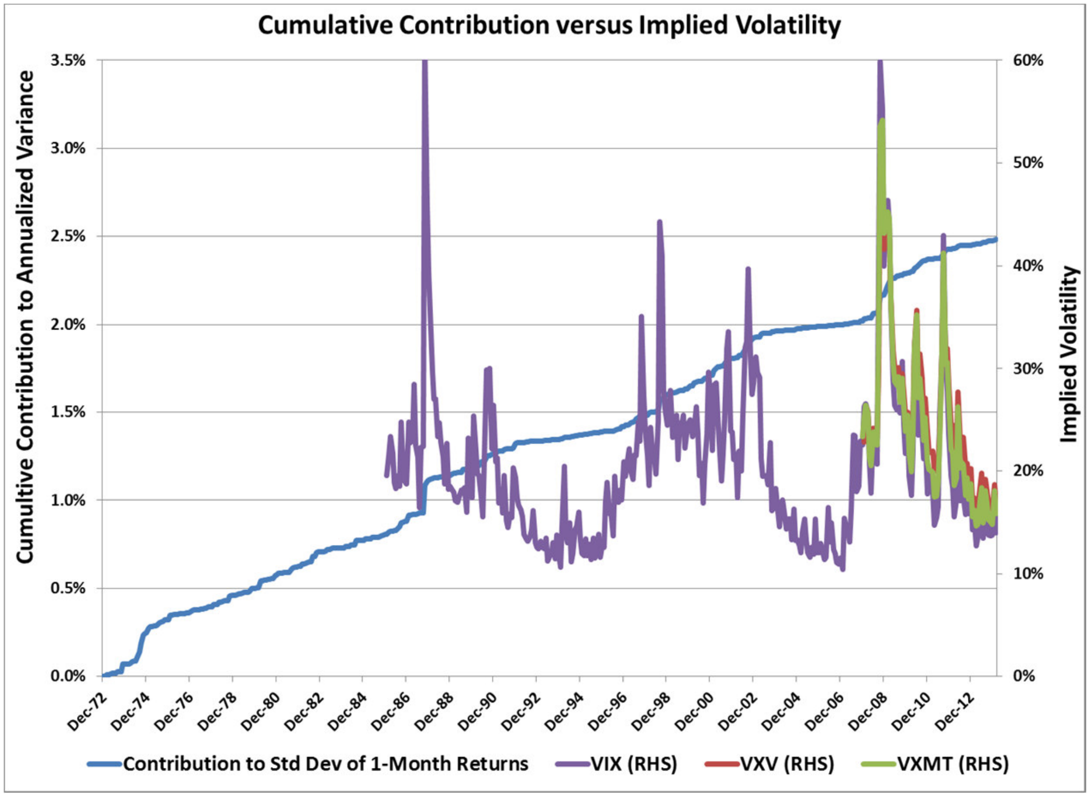
# Rolling volatility doesn't provide the same insights as cumulative contribution to variance chart.




Observations:

- Rolling volatility is affected by data leaving the sample.
- Rolling volatility doesn't identify start and end points of volatility regimes.
- Rolling volatility spiked after October 1987, but fell 36 months later.
- Rolling volatility ranged from 8% to 22%. Cumulative contribution to variance showed more stable behavior.

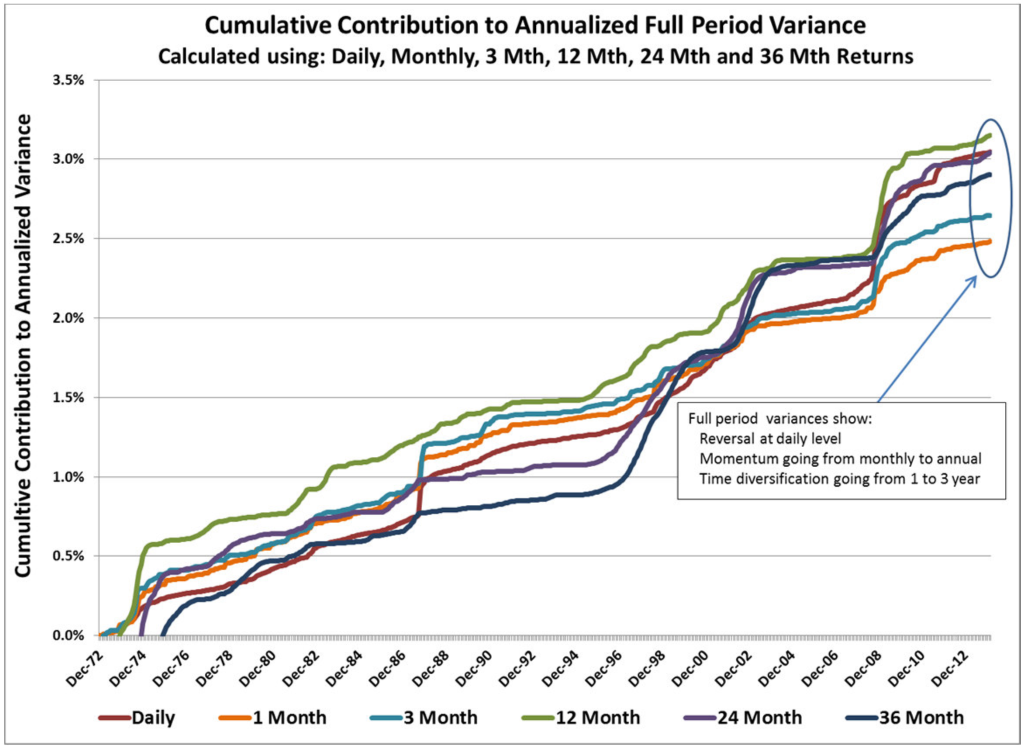
# Implied volatility doesn't provide the same insights as the cumulative contribution to standard deviation.




Observations:

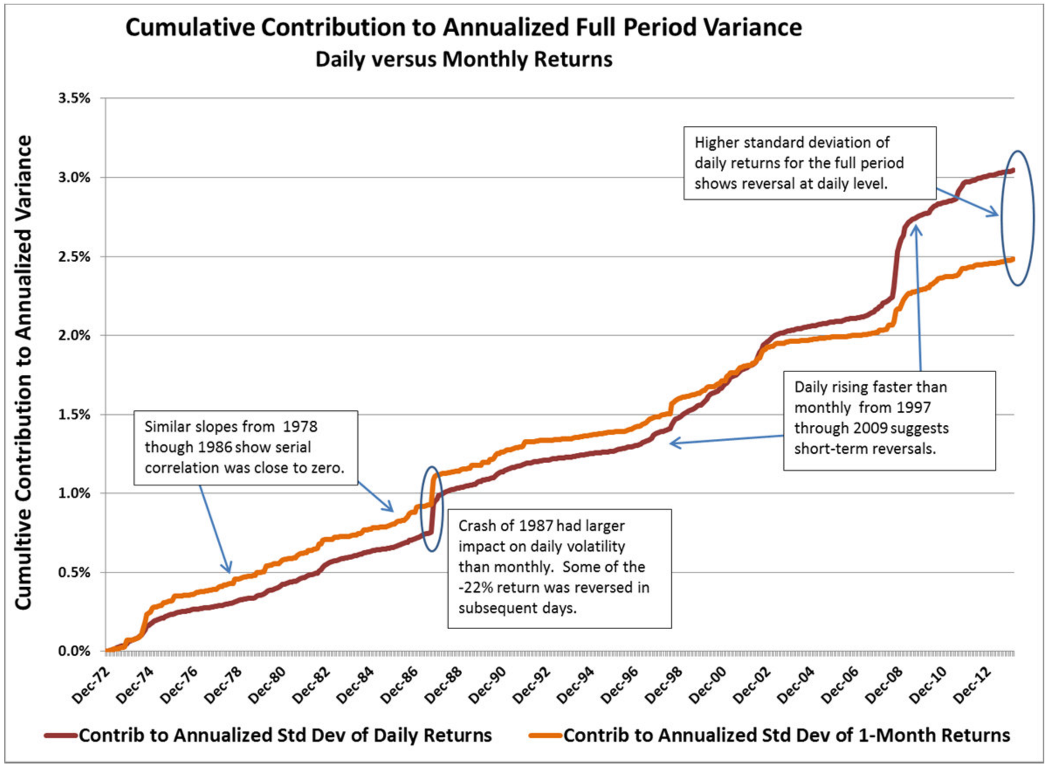
- Implied volatility moves more due to changes in the price of volatility (cost of insurance) than changes in expected volatility.
- Implied volatility for all 3 horizons spiked above 30% in response to market moves.
- Cumulative contribution to variance of monthly returns showed more stable behavior. More appropriate for investors with longer horizons.

# Cumulative contribution to variance charts show the effects of serial correlations and/or pace of news arrival.




- Variance increasing with holding period
  - Positive serial correlation (momentum / under reaction)
  - Some types of news arrives at lower frequency.
- Variance decreasing with holding period.
  - Negative serial correlation (time diversification / over reaction / reversal)

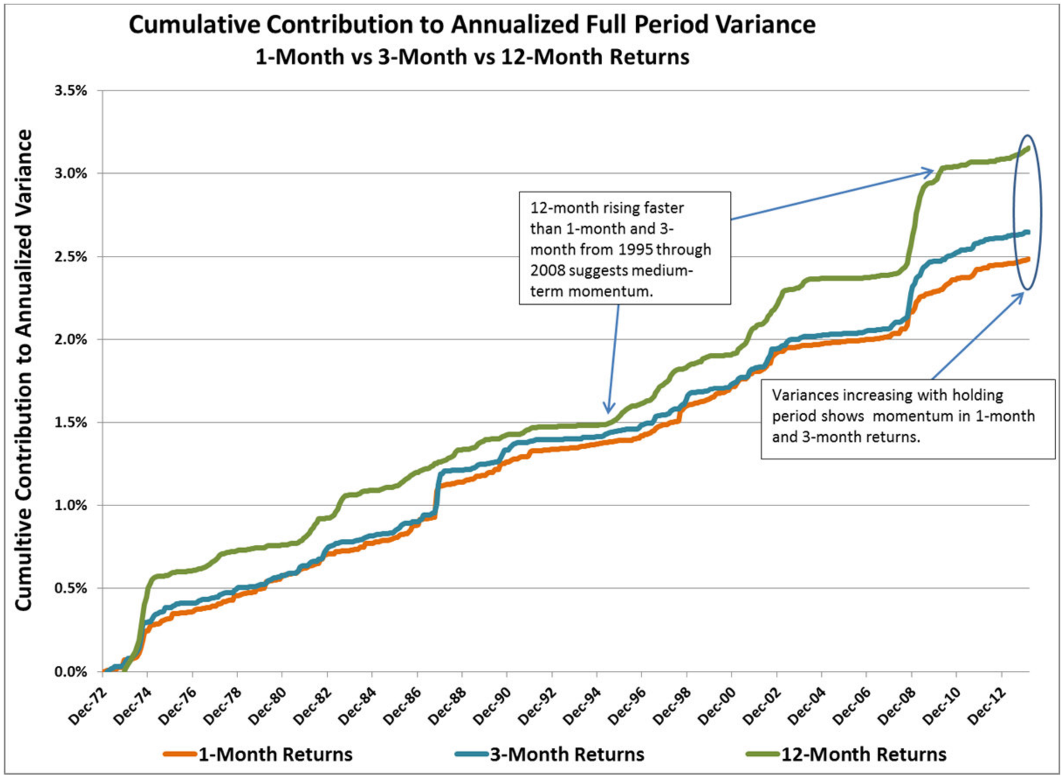
# Volatility of daily returns has been higher than monthly returns since the mid 1990s.




Observations:

- Higher standard deviation of daily returns for the full period shows reversal at daily level.
- From 1997 through 2013, daily returns line is steeper than 1-month. Volatility of 20.5% versus 16.0%.
- From 1974 though 1997, other than 1987 Crash, similar slopes for daily and 1-month lines. Little serial correlation in daily returns during that period.
- *Future will look like period since 1997. Monthly vol will be less than daily vol. Model risk using monthly returns.*

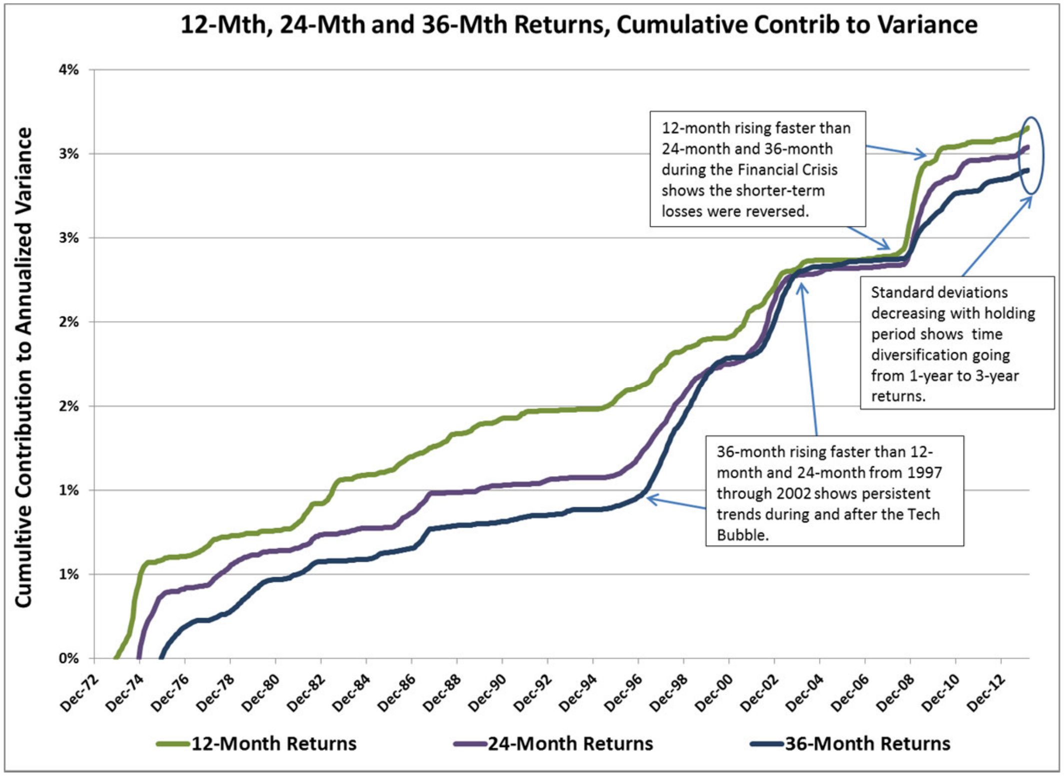
# Higher variance of 3- and 12-month returns shows positive serial correlation or impact of lower frequency news.




Observations:

- Full period variances increased with holding period, reflecting positive serial correlation (momentum) in 1-month and 3-month returns.
- Most of the positive serial correlation at the 12-month horizon occurred from 1995 through 2008.
- Similar slopes of lines for 1-month and 3-month returns suggests that the positive serial correlation of 1-month returns was episodic.
- *Should not use monthly returns to forecast annual volatility without an adjustment.*

# Data shows time diversification at multi-year horizons. Pattern was reversed during and after the Tech Bubble.




Observations:

- Time diversification at longer horizons. Variances decreased with holding period.
- Trending market during and after Tech Bubble appears as 36-month rising faster than 12-month and 24-month.
- Reversal of Financial Crisis losses appears as 12-month rising faster than 24-month and 36-month.
- *Assume some time diversification at 36-month investment horizon.*

# Volatility Summary

Volatility and serial correlations varied through time.

Full period behavior:

- Short-term reversals:
  Daily volatility was higher than monthly.
- Medium-term momentum:
  Volatility of 3- and 12-month returns was higher than 1-month
- Long-term time diversification:
  Volatility of 24- and 36-month returns were lower than 12-month.

But:

- Volatility of monthly returns was higher during the 1970s and 1980s.
- Positive serial correlation of annual returns during and after the Tech Bubble

# II. Cumulative contribution to correlation charts provide insights into the behavior of correlations over time.

Estimating correlations requires addressing the same issues as estimating volatility.

Cumulative Contribution to Correlation charts provide insights that assist in deciding:

- What holding period do we use for returns?
- What time period do we use to estimate the statistics?

# Calculating cumulative contribution to correlation

1. Correlation(Stocks, Bonds) =

$$\frac{\text{Covariance (Stocks, Bonds)}}{\text{Std Dev (Stocks)} \times \text{Std Dev (Bonds)}}$$

2. Covariance(Stocks, Bonds ) =

$$\frac{\sum((\text{Stk Ret}_{\text{t}} - \text{Avg Stk Ret}) \times (\text{Bnd Ret}_{\text{t}} - \text{Avg Bnd Ret}))}{\text{Total number of periods } - 1}$$

3. Cumulative Contribution to Correlation(Stock, Bond) =

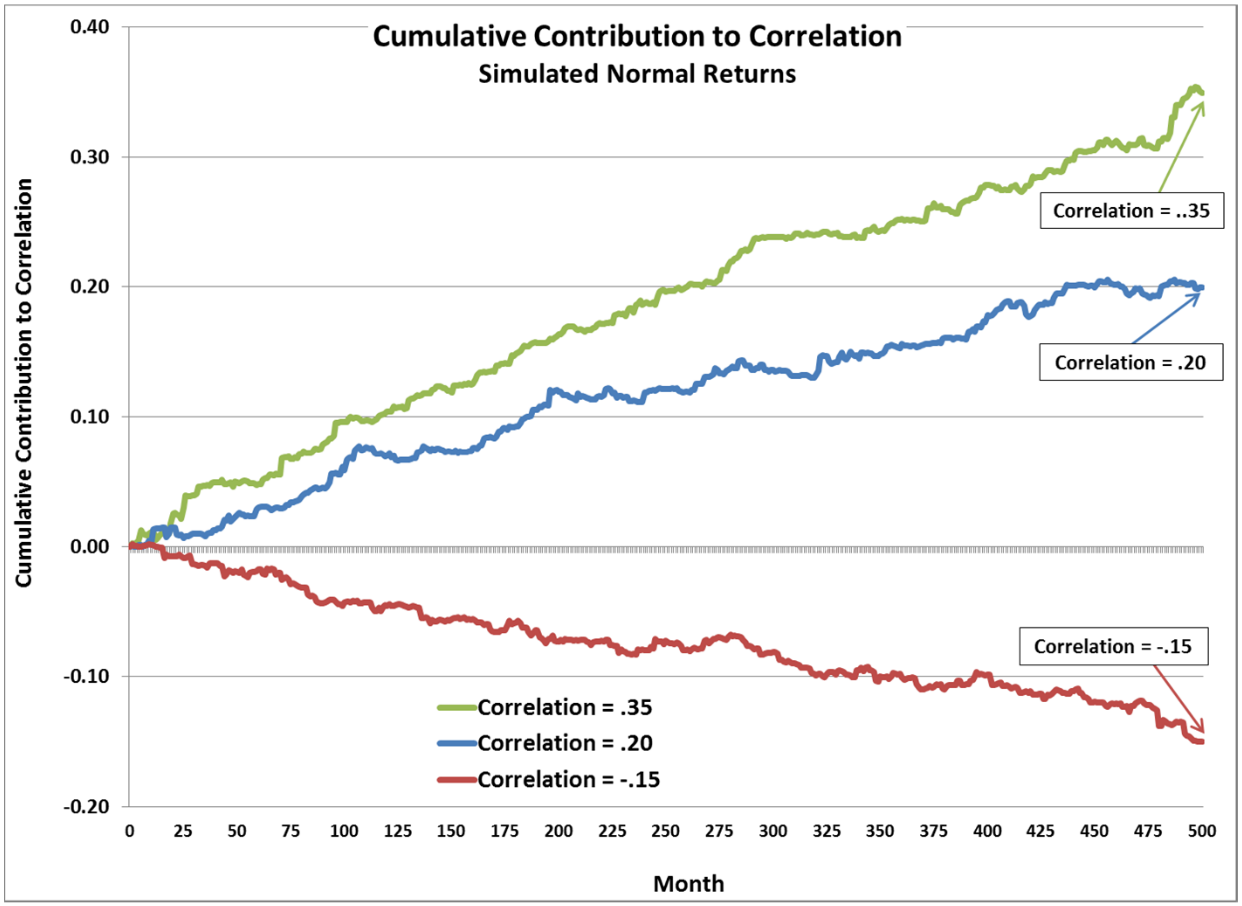
$$\frac{\sum_{1}^{\text{t}}((\text{Stock Return}_{\text{t}} - \text{Avg Stk Ret}) \times (\text{Bond Return}_{\text{t}} - \text{Avg Bnd Ret}))}{(\text{Total number of periods} - 1) \times \text{Std Dev(Stocks)} \times \text{Std Dev(Bonds)}}$$

Note: Covariances grow linearly with time, so contributions to correlations are linear as well.

# Interpreting Cumulative Contribution to Correlation Charts

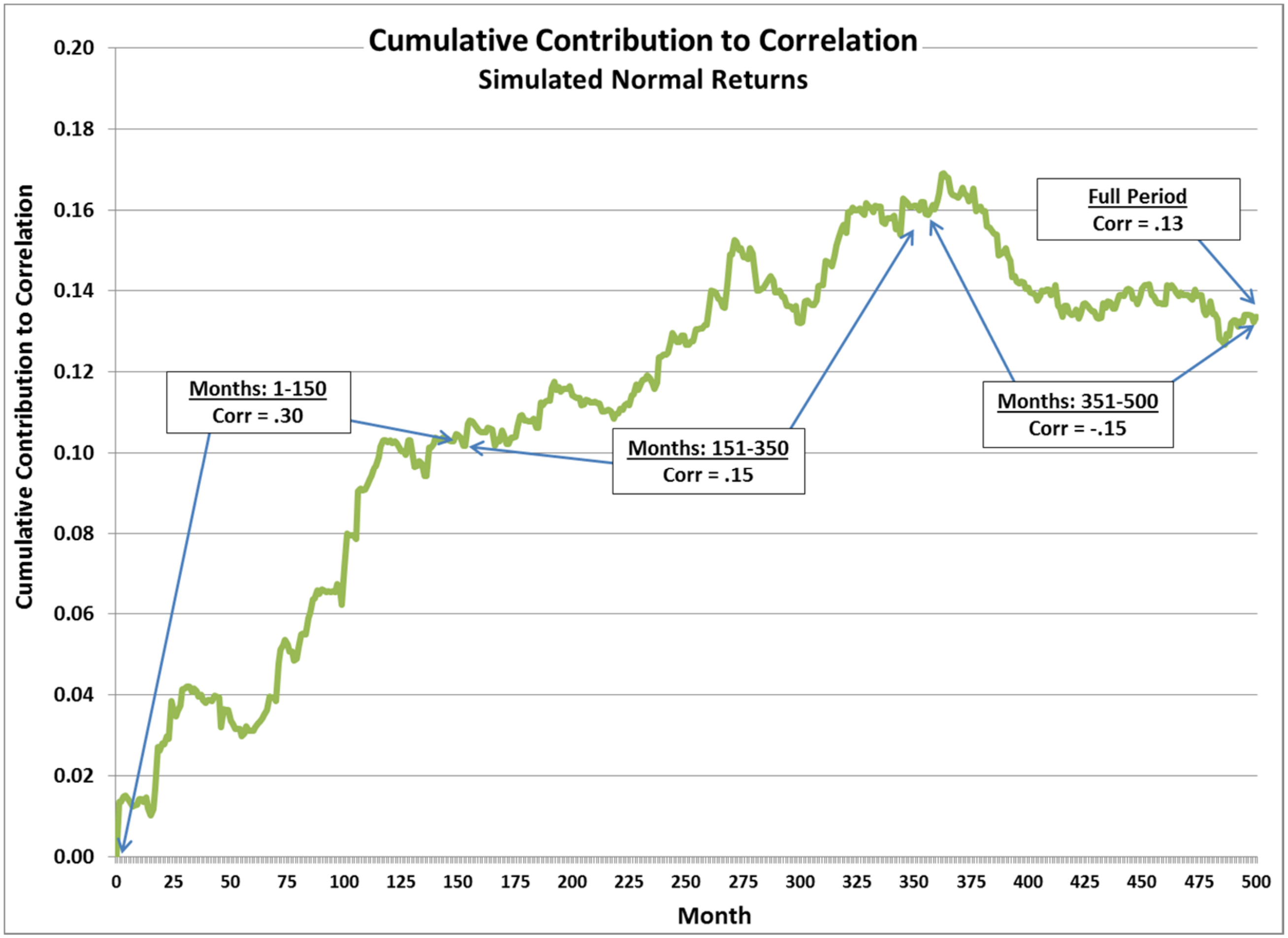
## Constant Correlation (simulated returns)

- Higher correlation results in steeper line.
- Negative correlation results in downward slope
- Lines end at full period correlation.



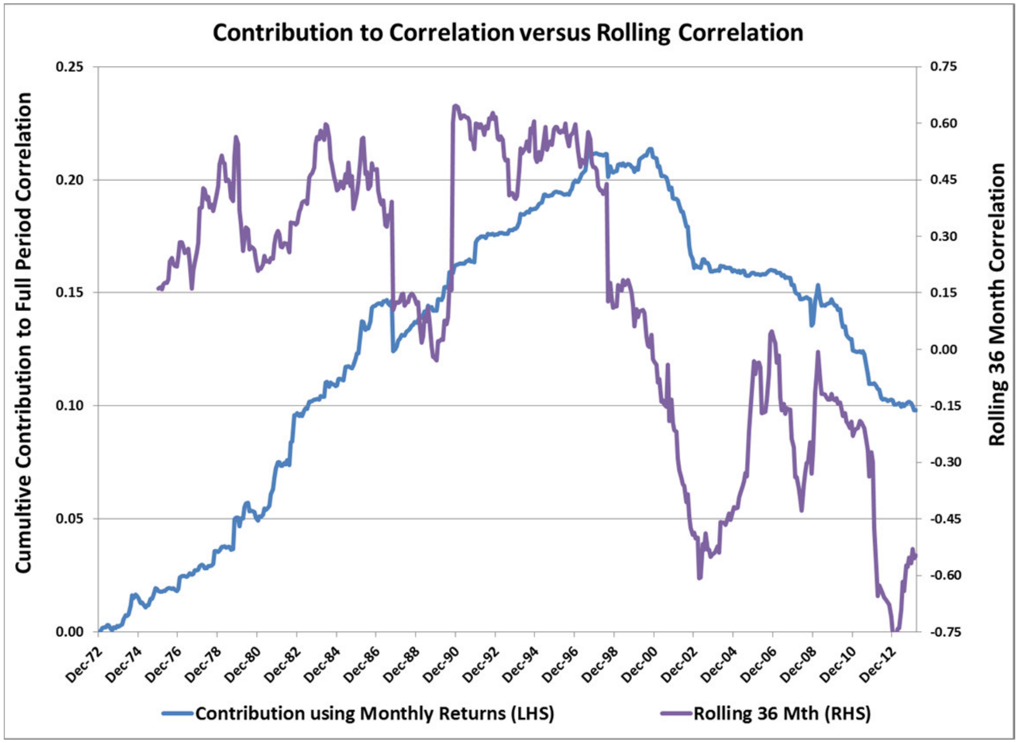
# Interpreting Cumulative Contribution to Correlation Charts
## Time Varying Correlation (simulated returns)

- Slope of line changes as correlation changes.
- Line ends at full period correlation.



# Cumulative contribution to correlation chart provides insights unavailable using full period and rolling window.

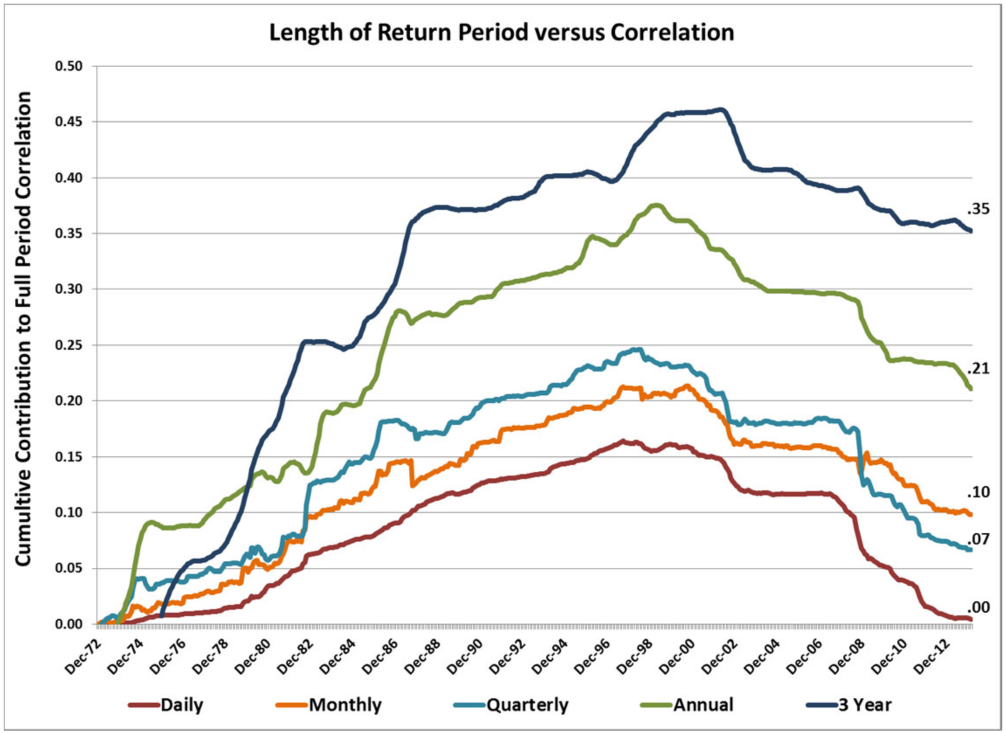
- Slope of cumulative correlation line shows the evolution of correlations through time.
- Cumulative contribution shows correlation of monthly stock and bonds returns changed from positive to negative Oct. 2000.
- Rolling correlation was zero in October 2000, and didn't get to -.30 until 2002.
- Cumulative correlation is not sensitive to data points dropping out of the sample (e.g. Oct. 1987)




| | Correlation of Monthly Returns |
|---|---|
| 1972 - 2013 | 0.10 |
| 12/1972 – 9/2000 | 0.31 |
| 10/2000 – 12/2013 | -0.37 |

# Stock versus Bond correlation increased with holding period.

- Investors should use a return period that's consistent with their investment horizon.
- The correlation calculated using 3-year returns was larger than correlations calculated using returns for shorter periods.
- Higher degree of correlation shows that there are regimes and cycles that are not captured in short-term returns.



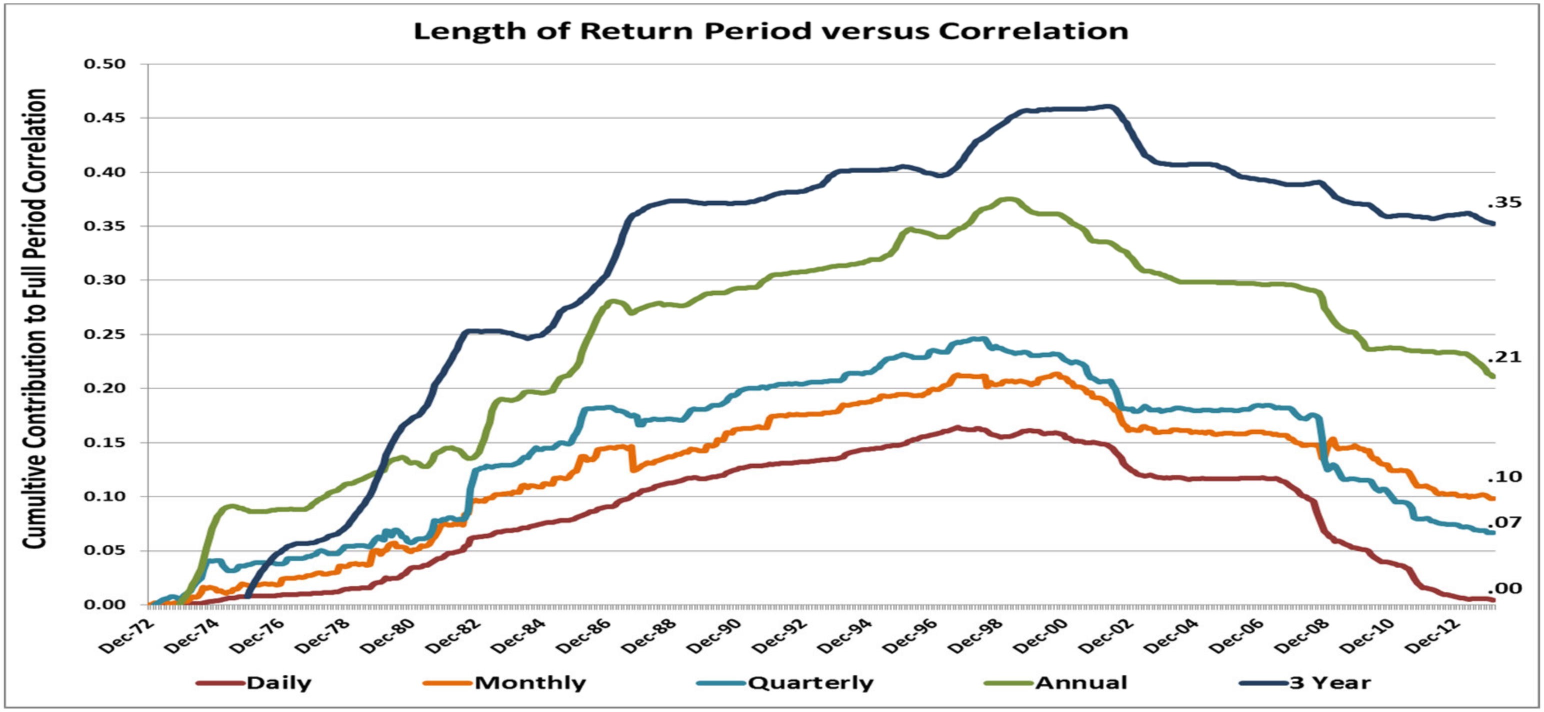
# Cumulative contribution to correlation allows us to identify turning points.

| | Holding Period for Return Calculation | | | | | |
|---|---|---|---|---|---|---|
| | Daily | 1-Mth | 3-Mth | Annual | 2-Year | 3-Year |
| Full Period: 1972 - 2013 | 0.00 | 0.10 | 0.07 | 0.21 | 0.27 | 0.35 |
| While Positive | 0.32 | 0.31 | 0.38 | 0.53 | 0.60 | 0.62 |
| While Negative | -0.34 | -0.37 | -0.52 | -0.65 | -0.71 | -0.71 |
| Date Changed from Positive to Negative * | Oct-97 | Sep-00 | Mar-98 | Apr-99 | Sep-99 | Apr-02 |

- The correlations calculated using annual returns are roughly 1½ times the correlations calculated using monthly returns.
- The correlations calculated using 3-year returns are roughly two times as large.

* Date of peak. All of the correlations turned negative at approximately the same time.
Much of the difference can be attributed to slower reaction of longer holding period returns.
E.g. The return for January 2000 is in the 3-year return through December 2002.

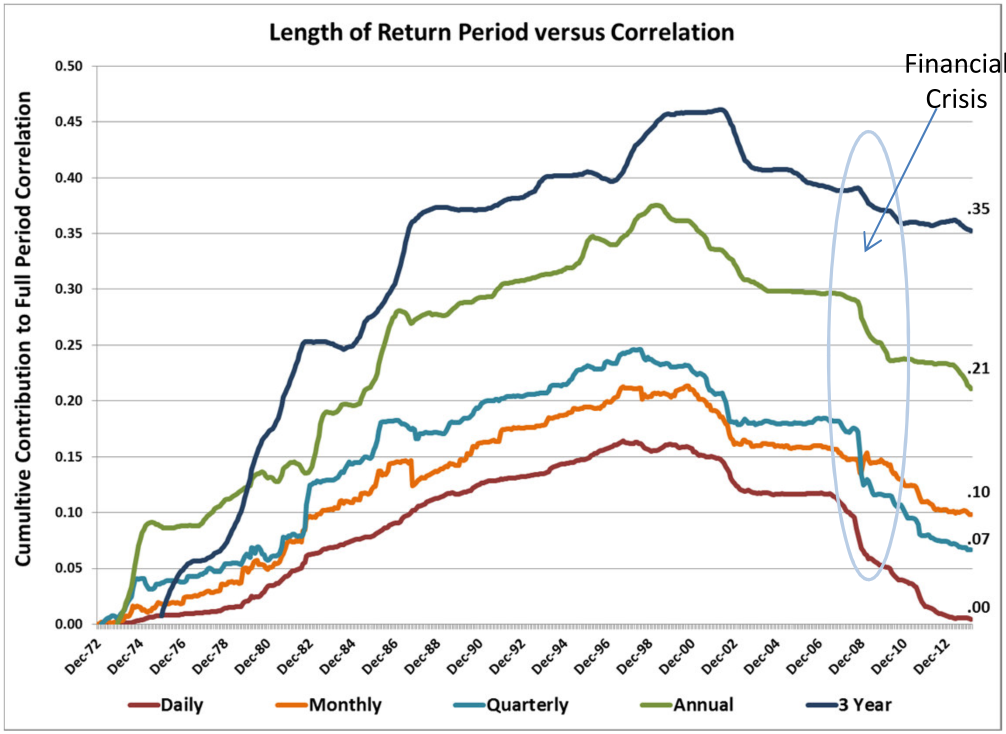
# Stock versus Bond correlation is regime dependent.




| | Macro Environment | Correlation |
|---|---|---|
| 1970s | Changing inflation expectations | + |
| 1980s | Falling interest rates | + |
| 1990s | Capital flows into both stock and bond markets | + |
| 2000s | Tech Bubble and Financial Crisis result in interest rates responding to growth surprises. | - |

# Changes in the slope of lines highlights regimes

- The steep 3-year line between 1975 and 1981 shows that the environment of highly volatile inflation expectations translated into high stock-bond correlations for longer-term investors.
- There wasn't much variability in correlations between 1972 and 2000 using daily and monthly returns.
- The correlations were more negative after the Tech Bubble and Financial Crisis than during the period between.



# Correlation Summary

Charting time series contribution to correlation provides insights that are obscured by rolling windows and other approaches.

1. Correlation between stocks and Treasuries was regime dependent.
   - Positive through the Tech Stock Bubble
   - Negative since
2. Correlation between US stocks and Treasuries increased with the holding period.
   - More positive when positive
   - More negative when negative

# Discussion

- We use charts of cumulative returns to understand performance, why shouldn't we use similar tools to understand risk?
- While the presentation focused on the visualizing asset class risks and correlations, similar cumulative contribution charts could be used to visualize other types of risks such as serial correlation, tracking error and information ratios.