MOODY'S ANALYTICS

Modeling Sovereign Correlations

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Introduction

A critical concern for a credit portfolio containing sovereign debt is the correlation of sovereign credit risk with other instruments in the portfolio.

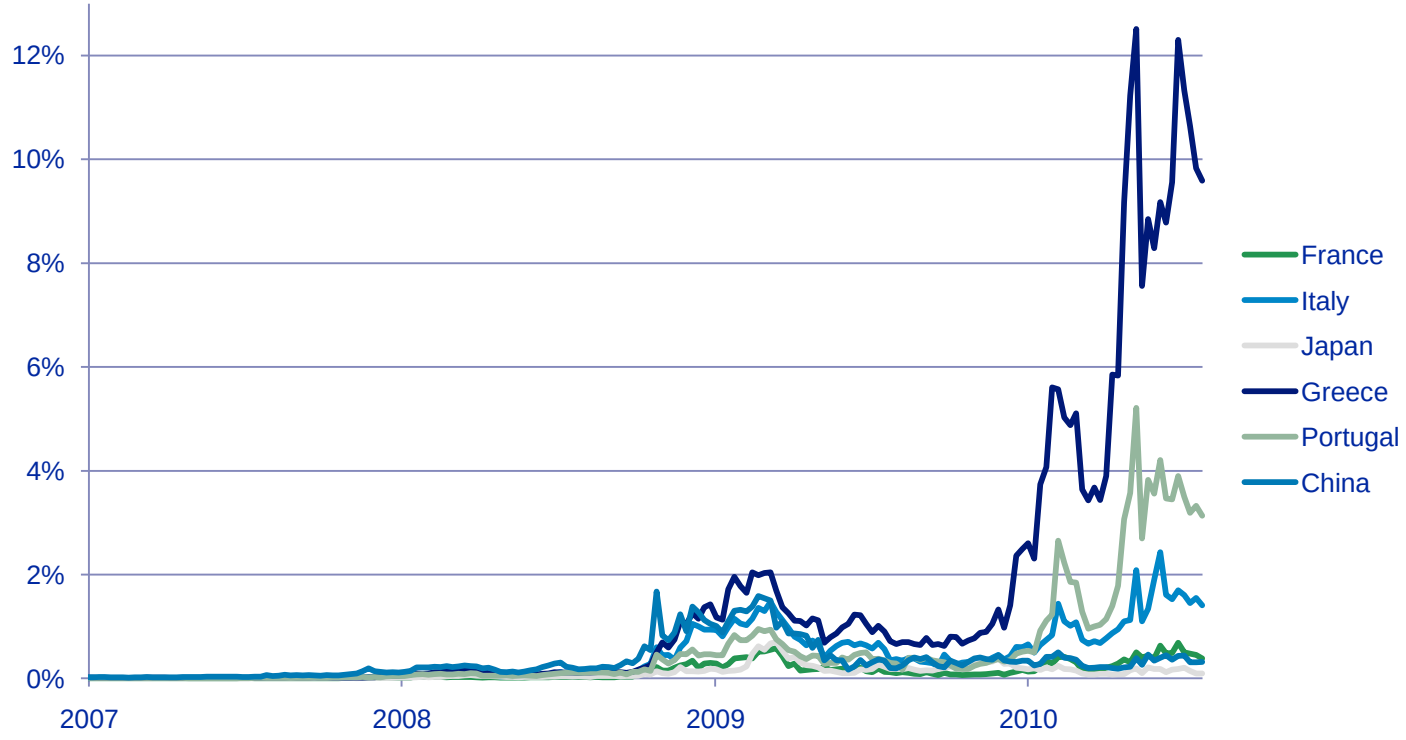
We devise methodology that relies on **CDS market data** and **GCorr**, Moody's Analytics asset return factor model to estimate the asset **correlations** between:

- Any two sovereign nations
- A sovereign nation and an entity from another asset class including:
  - 42,000 public firms
  - Private firms
  - CRE exposures
  - Retail exposures

Once calculated, these correlations can serve as inputs to a portfolio optimizer, a portfolio risk system, or a bond valuation engine.

The Sovereign Credit Spreads Move in Tandem

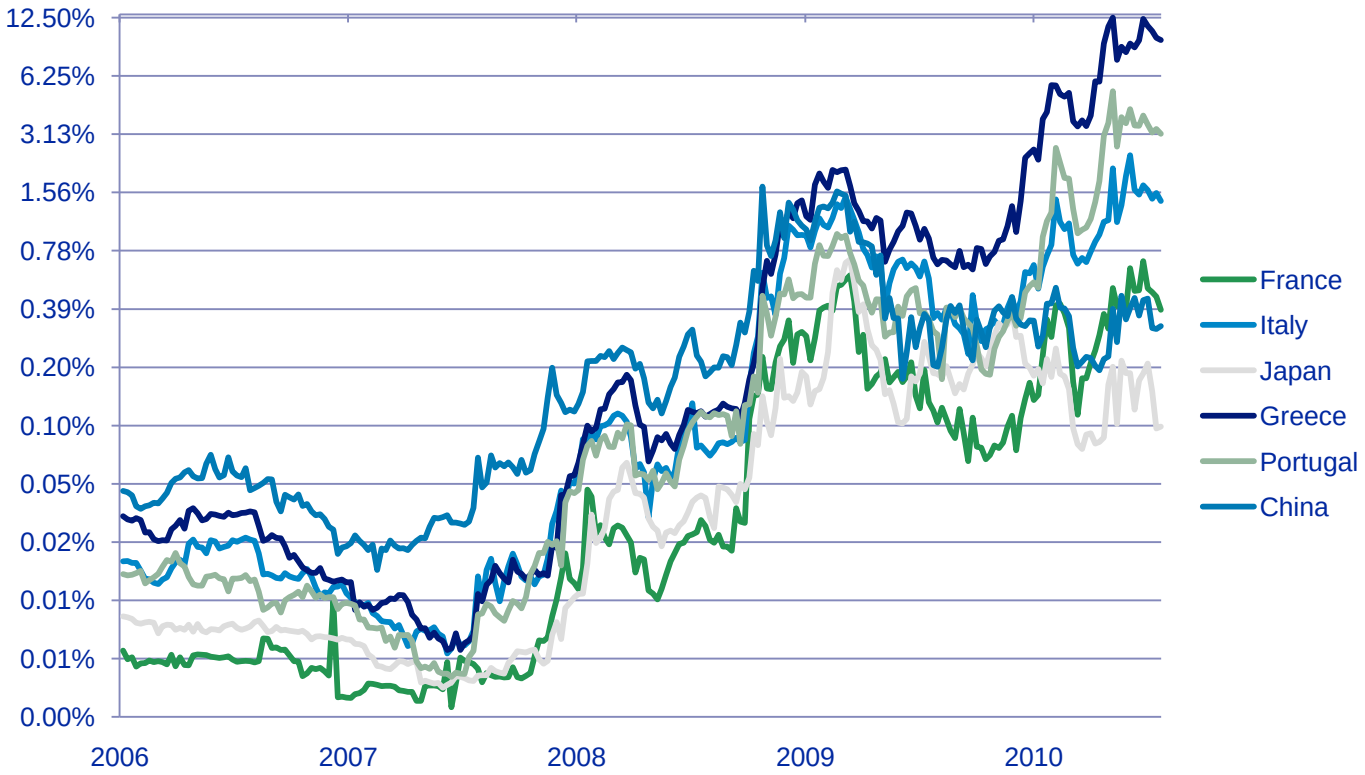
Sovereign CDS Spreads: Jan. 2006-Present



1y cumulative probability of default implied by the CDS spread

The Sovereign Credit Spreads Move in Tandem

Sovereign CDS Spreads: Jan. 2006-Present (Log. Scale)



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Strategy for Capturing Correlations

Factor Model Approach:

- We integrate Sovereigns into a Factor Correlation Model by using the concept of R-squared (i.e., RSQ)
- Sovereign RSQs capture how a sovereign's credit comoves with the sovereign's country and region fundamentals (i.e., RSQ is portion of variation in sovereign credit explained by fundamentals)
- We define "**fundamentals**" as the asset returns of corporate firms
- Methodology accounts for "**sovereign contagion**" effects via factor definition

Key Empirical Findings

- Sovereigns are more correlated with each other than with corporates
  - Two sovereigns in different regions are often more correlated than the corporate firm assets indexes for the two countries
- Sov. RSQs are spread out over sizeable range
  - Countries vary to the extent their sovereign risk is tied to fundamentals
- Similar countries have similar Sov. RSQs
- Validation results
  - The methodology can be used for calculating the correlations both between the sovereigns and sovereigns and corporates in a factor model framework

Spread Implied Correlations

Use CDS market to back out the Spread-implied correlation of **asset returns**:

Actual CDS spreads → #1 → Risk Neutral Prob. (Q) → #2 → CDS-implied EDF → #3 → CDS-implied *DD* → #4

$$\rho_{j,i} = corr(\Delta DD_j^{CDS}, \Delta DD_i^{CDS}) = corr(r_j^{SOV}, r_i^{SOV})$$

Step #1: Use the CDS spreads to calculate the risk neutral probabilities of default

Step #2: Convert risk neutral probabilities into SI-EDF (Spread-Implied Expected Default Frequency) using market risk premium

Step #3: Distance-to-Default (i.e., DD) is a transformation of SI-EDF

Step #4: In Black-Scholes-Merton framework, correlation of DD changes equals to asset return correlations.

Exploratory Analysis: Sovereign Correlations During Financial Crisis

Analysis: Take a sovereign (e.g., Austria), and calculate:

- Average correlation of Austria with all other sovereigns
- Average correlation of Austria with 300 biggest European and US corporates

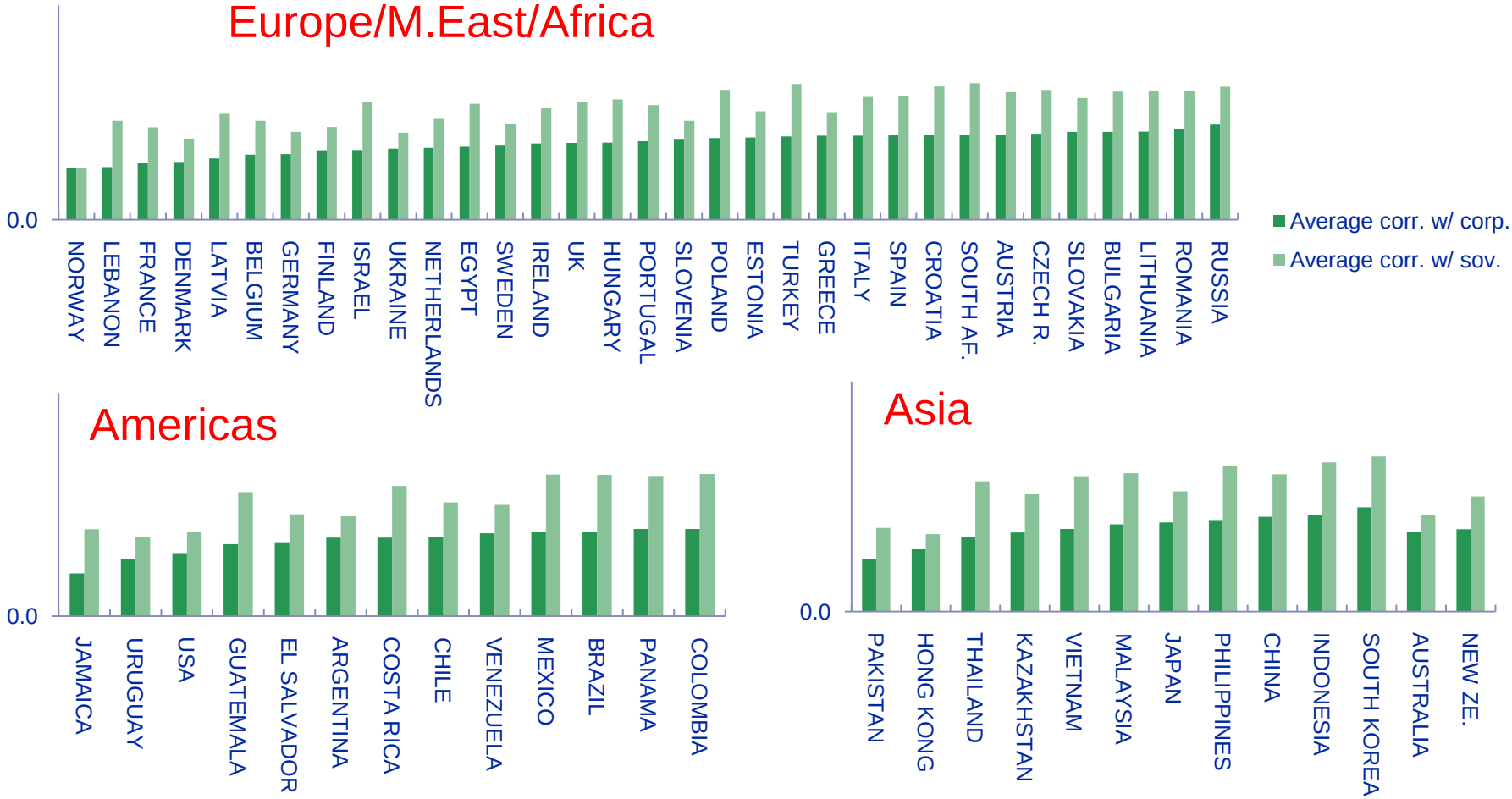
Finding: Level of correlations

- Sovereigns tend to be correlated more with other sovereigns than with corporates
- Two sovereigns in different regions (e.g., Austria and Malaysia) are often more correlated than their countries' corporate asset indexes

Finding: Rank ordering of correlations

- Sovereigns highly correlated with other sovereigns also tend to be correlated with corporates
- Therefore, sovereign and corporate markets capture similar information

Average Correlation of a Sovereign with Other Sovereigns and Corporates During Crisis



Time period of analysis: [June 2008, June 2010]

Why do we Need a Factor Model?

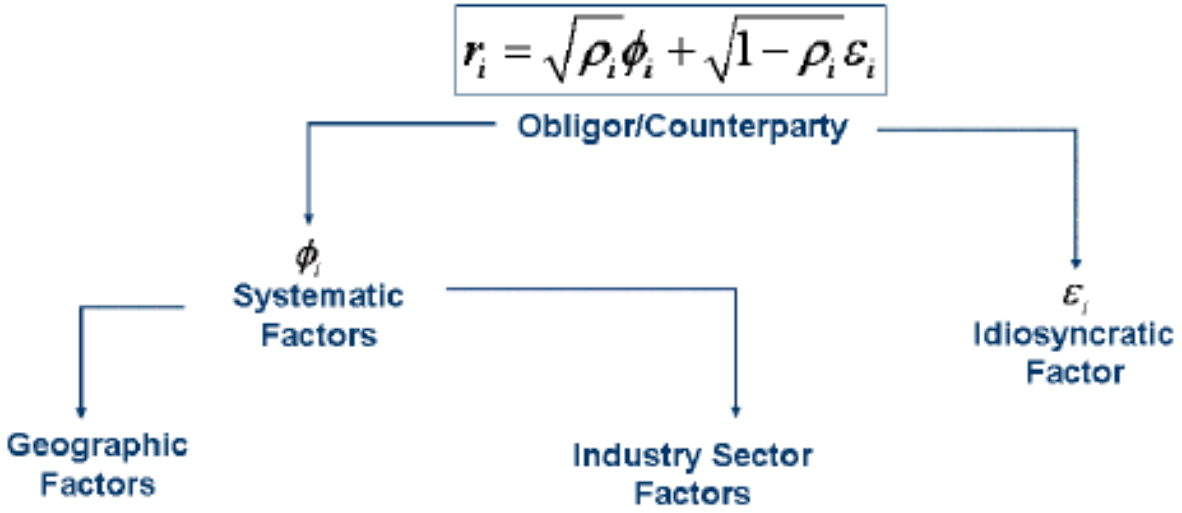
Calculating Spread Implied Correlations using ONLY spread data requires that both entities *i and j* have traded CDS spreads

- However, only about 1000 European and N. American firms have liquid spreads

Our methodology leverages GCorr factor structure in addition to CDS spreads, enabling us to calculate correlations between a sovereign nation and an entity from another asset class including:

- 42,000 public firms
- Private firms
- CRE exposures
- Retail exposures

GCorr Model Factor Structure



- GCorr is a structural, multi-factor based correlation model
- GCorr breaks up an asset return into systematic and idiosyncratic component
- Systematic return is decomposed into **49 Country** and **61 Industry** components
- For a firm *i,* composite index ϕ_i is determined by its country and industry
- ρ_i (i.e., RSQ) is the % of variation in firm returns explained by index ϕ_i

GCorr Correlation: Application to Sovereigns

GCorr correlation between **firms** i and j is given by:

$$corr(r_j, r_i) = \sqrt{RSQ_j}\sqrt{RSQ_i}\, corr(\phi_j, \phi_i)$$

where:

ϕ_j is a composite index for firm j, determined by its industry and country

RSQ_j is a percentage of variation in j's asset returns explained by ϕ_j

For **sovereigns**, we define:

ϕ_j : Constructed by using representative corporate asset returns for sovereign's country ***and*** region

RSQ_j : estimated from the spread-implied correlations

Sovereign ϕ_j Construction: Economic Motivation

Sovereign ϕ_j defined using the regional factors (in addition to country factor) to take into account:

- Common Currency effects:
  - For example, Portugal's credit depended on the willingness of the larger EU members to bail it out
- Regional credit contagion effects
  - The tendency of a crisis in one sovereign country to spared out to its immediate neighbors (e.g., South Asian crisis, Latin American crisis etc.)

Sovereign RSQ: Interpretation

What Sovereign RSQs seek to capture:

R-squared captures the extent to which changes in country's credit worthiness reflect changes in the sovereign's country and region fundaments.

Other potential drivers of credit risk:

- Flight to quality: even if US firms deteriorate, US T-bills still could be perceived as "safe," compared to other asset classes; funds flow to US bonds, driving the spreads down
- Some countries can inflate their way out of debt, while others cannot (e.g., EMU)
- A country could have large cash reserves making its changes in spreads unrelated to the changes in fundamental values (e.g., Norway)

Sovereign RSQs: Estimation Methodology

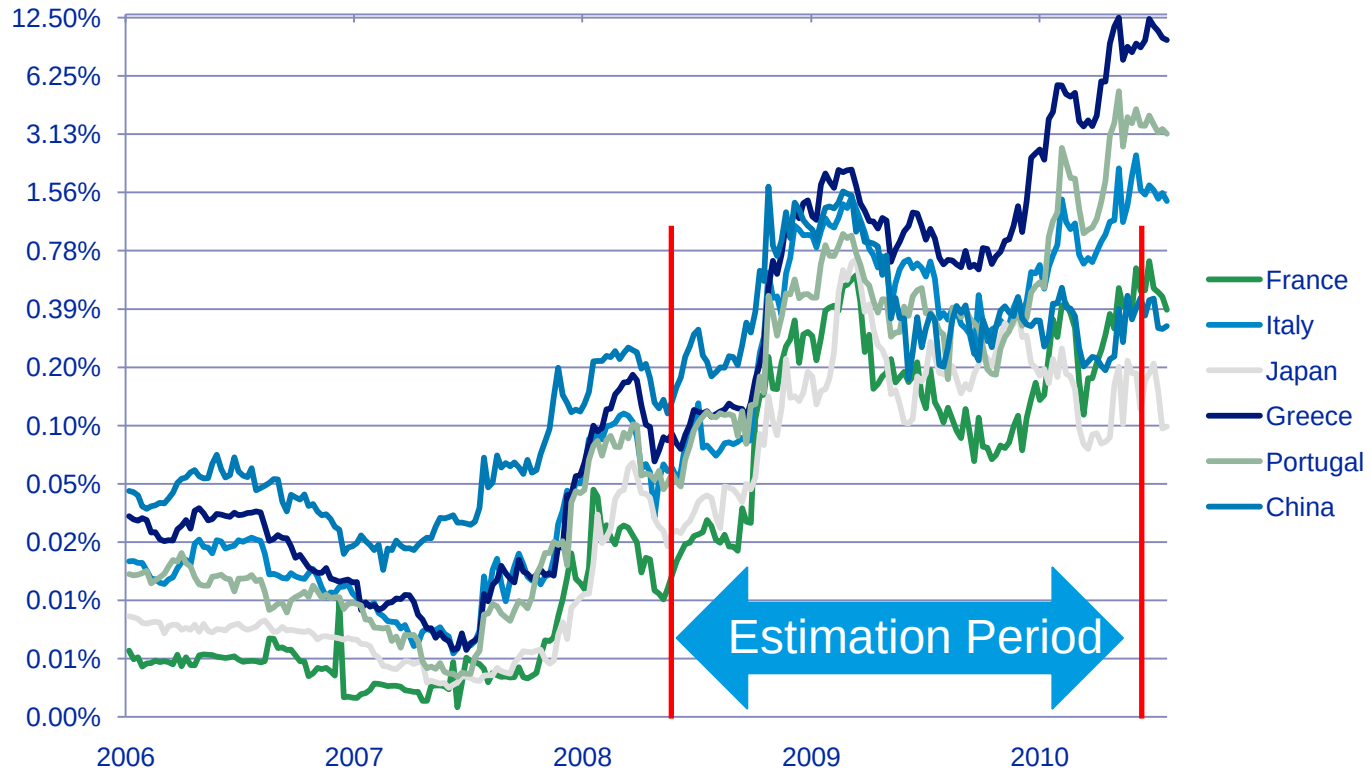
Sovereign CDS market provides us with the way to calculate sovereign asset correlations $corr\left(r_i^{SOV}, r_j^{SOV}\right)$

For each pair of sovereigns, we have:

$$\underbrace{corr\left(r_{USA}^{SOV}, r_{GER}^{SOV}\right)}_{KNOWN\ \ (CDS\ \ DATA)} = \underbrace{\sqrt{RSQ_{USA}}\sqrt{RSQ_{GER}}}_{NEEDS\ \ TO\ \ BE\ \ ESTIMATED}\underbrace{corr\left(\phi_{USA}, \phi_{GER}\right)}_{KNOWN\ \ (GCorr)}$$

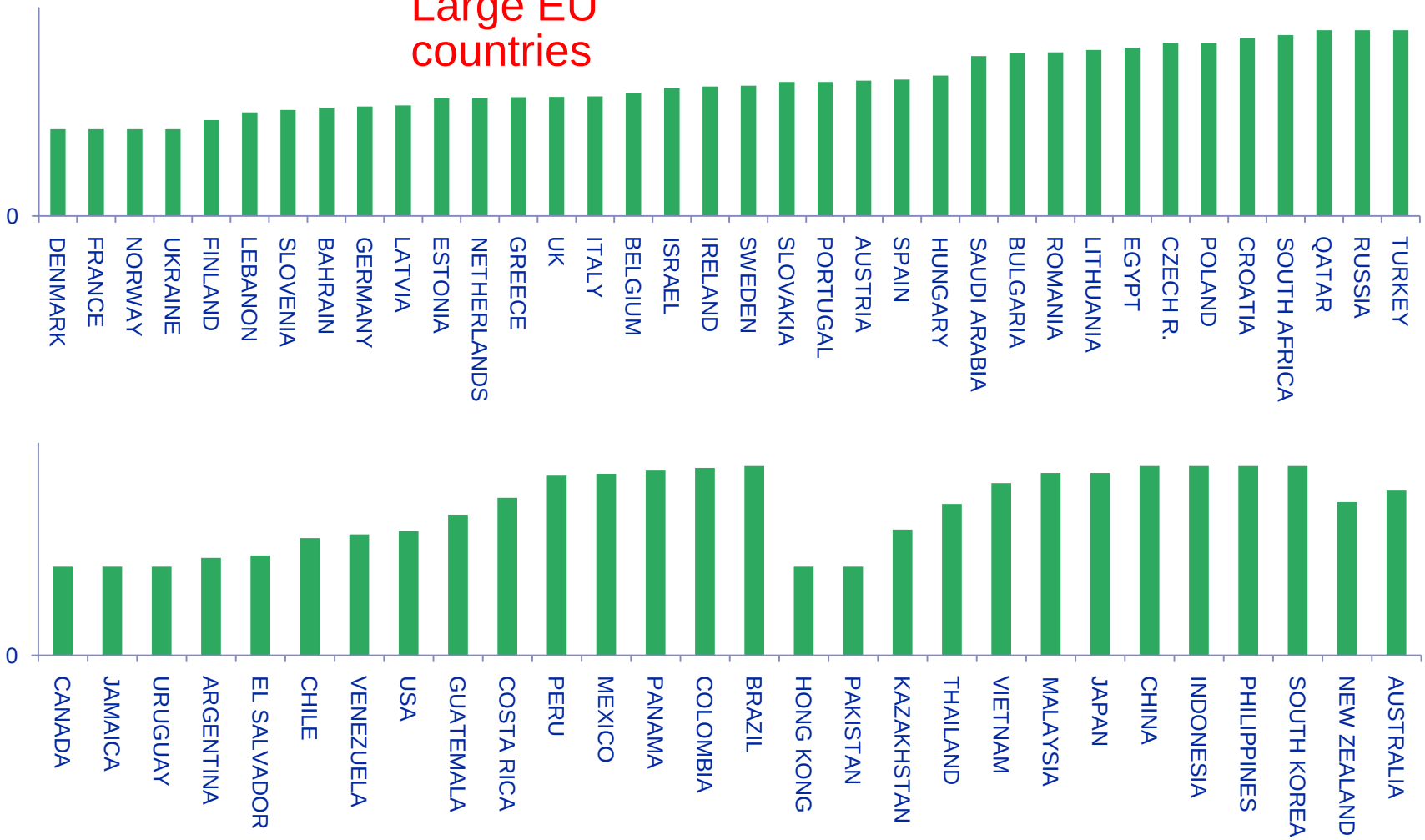
We use OLS to calculate R-squareds (i.e., RSQ_i):

Sovereigns: Estimation Timeframe [Jun 2008-July 2010]



1y cumulative probability of default implied by the CDS spread

Similar Countries Have Similar RSQs



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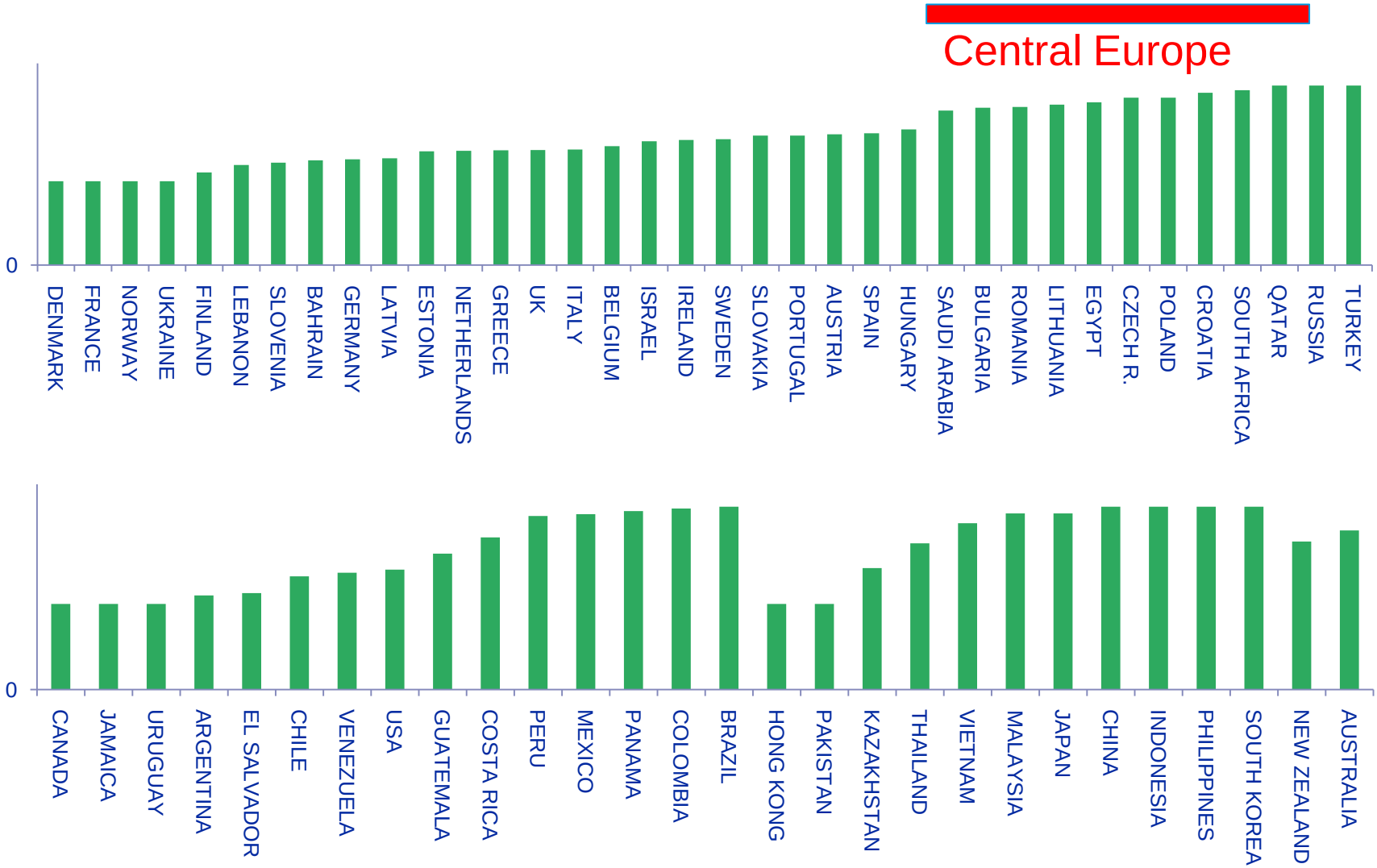

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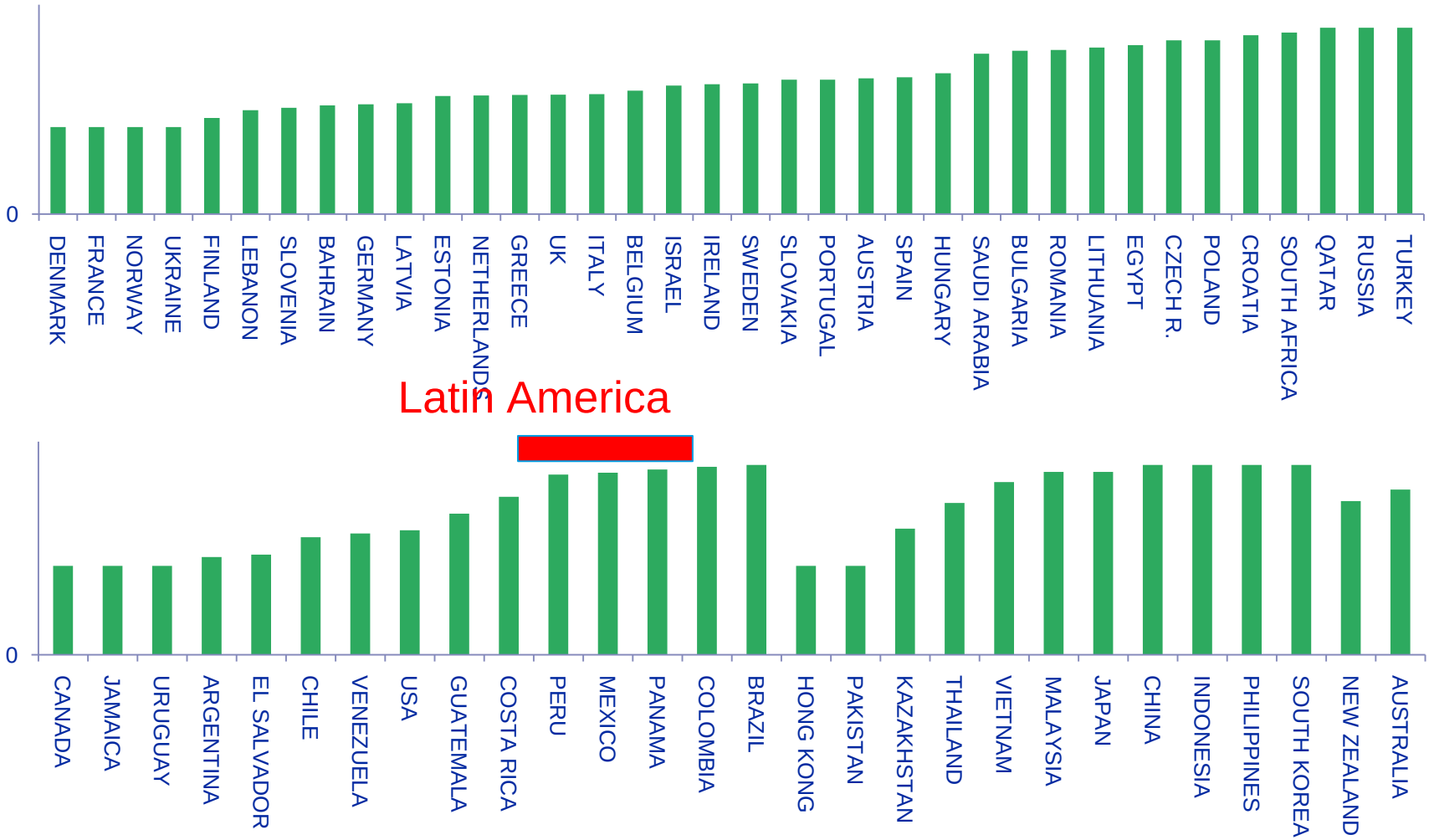

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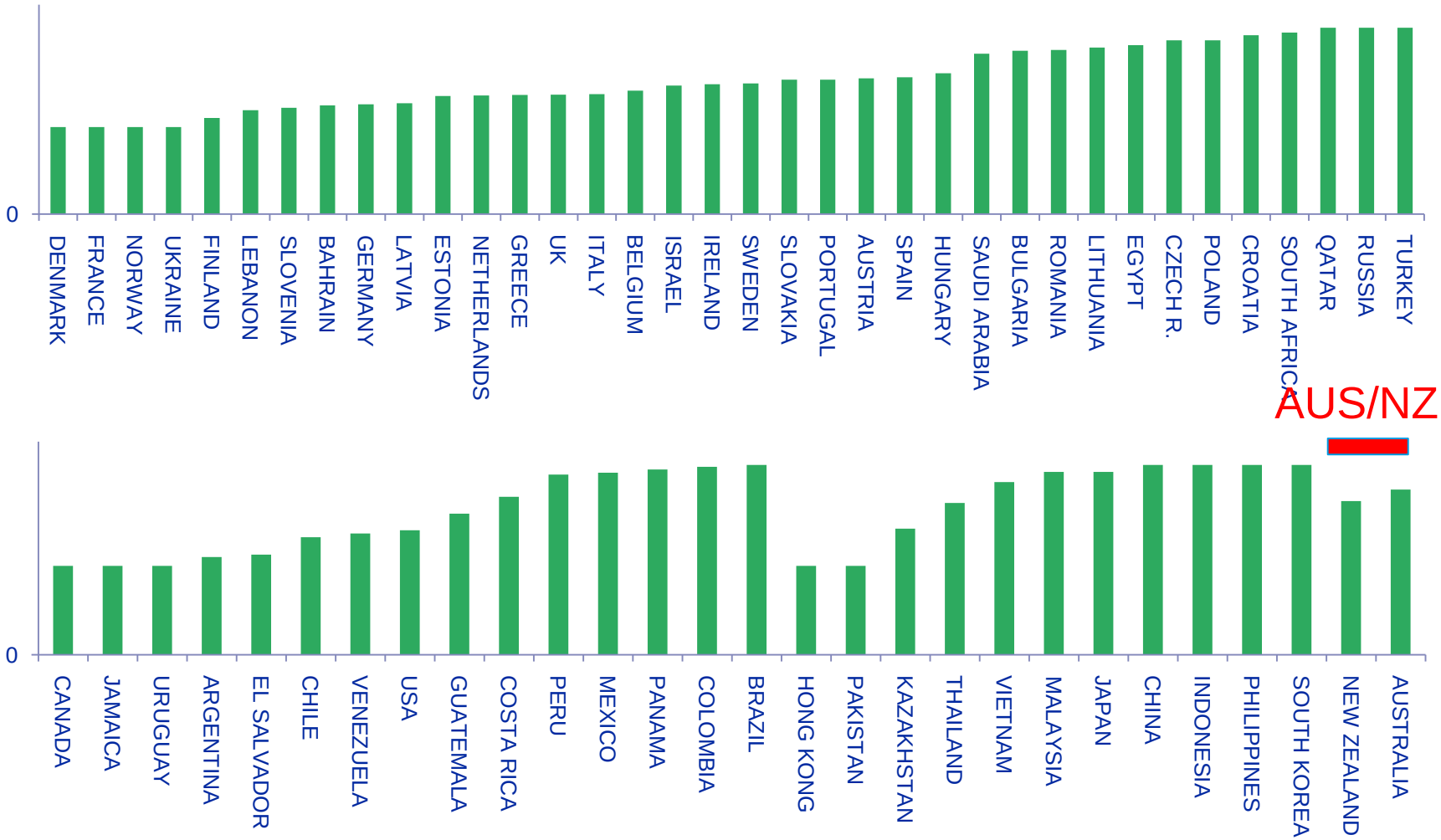
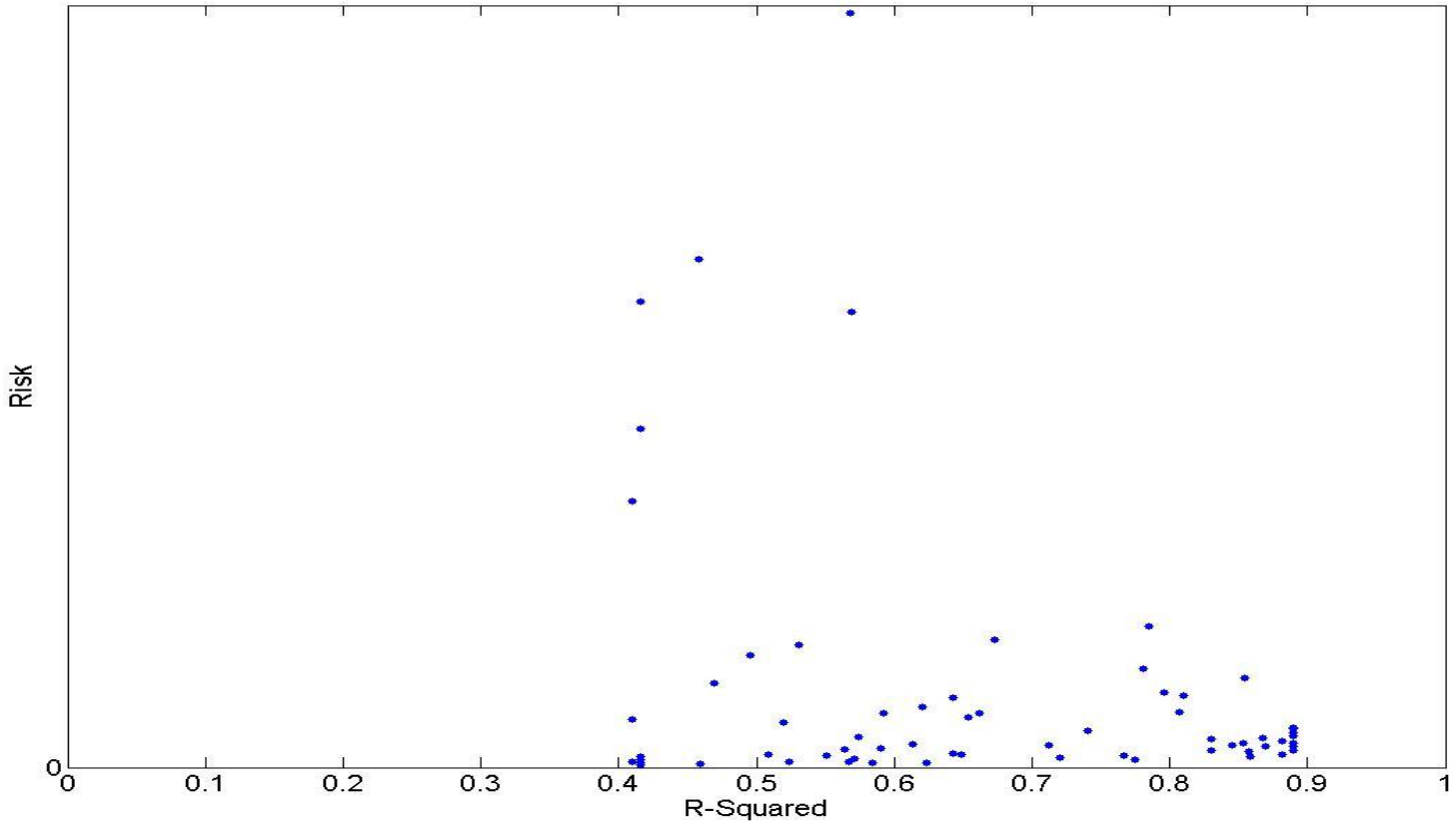


Sovereign RSQ vs. Risk Plot



RSQ-s are spread uniformly over a wide range: sovereign credit differs in its sensitivity to the underlying country fundamentals

Sovereign RSQ Validation

We validate our RSQ estimates by using 3 methods:

- Test #1: Cross-Sectional Fit of Predicted and Empirical Asset Correlation
- Test #2: Out-of-Sample Comparison with Corporate CDS
- Test #3: Including Corporate CDS directly into OLS estimation

#1 tests an implication of the postulated model

#2 and #3: "Will our sovereign correlation estimates perform well in a portfolio consisting of both sovereigns and corporate exposures?"

RSQ Validation (1) : High Level of Cross-Sectional Fit of Predicted and Empirical Correlations

Across the sovereign pairs, average levels of predicted,

$$\sqrt{RSQ_j}\sqrt{RSQ_i}\,corr(\phi_j,\phi_i)$$

and empirical correlations

$$corr\,(r_j^{SOV},r_i^{SOV})$$

match by construction.

To test the fit, we calculate the pair-wise correlation between the empirical and predicted estimates for each pair:

$$CORR\;\;((EMPIRICAL\;CORR\;\;i,j),(PREDICTED\;CORR\;\;i,j))$$

RSQ Validation (1) : High Level of Cross-Sectional Fit of Predicted and Empirical Correlations

Subsample of Safest Country Pairs:

Correlations	1	2
1: Empirical Corr.	100%	
2: Modeled Corr.	75.0%	100%

Means	
Mean Corr.	45.7%
Mean Corr.	47.7%

Overall Sample of Country Pairs:

Correlations	1	2
1: Empirical Corr.	100%	
2: Modeled Corr.	77.6%	100%

Means	
Mean Corr.	51.9%
Mean Corr.	52.3%

RSQ Validation (1) : Empirical and Modeled Correlations Closely Match Across Regions

Regions	1	2	3	4	5
Region 1 (Australia and New Zealand)	72.9% 74.1%				
Region 2 (North America – USA and Canada)	37.4% 41.1%	34.9% 49.1%			
Region 3 (Europe)	47.1% 51.8%	34.0% 42.9%	55.2% 60.2%		
Region 4 (LA EA ME Af)	43.8% 44.3%	31.3% 35.3%	48.3% 45.1%	62.5% 65.2%	
Region 5 (Japan)	52.1% 47.9%	28.3% 34.5%	49.6% 46.3%	55.0% 42.9%	100.0% 100.0%

Empirical Corr.

Modeled Corr.

Average $corr(r_j^{SOV}, r_i^{SOV})$ for a pair of regions

Average $\sqrt{RSQ_j}\sqrt{RSQ_i}\,corr(\phi_j, \phi_i)$ for a pair of regions

RSQ Validation (2) : Out-of-Sample Comparison

Validation of RSQ data by using corporate/ sovereign pairs

» Consider:

$$corr\ \left(\sum_{j=1}^{N_{CORP}} corr\left(r_{i,SOV}^{CDS}, r_{j,CORP}^{CDS}\right), \sum_{j=1}^{N_{SOV}} corr\left(r_{i,SOV}^{CDS}, r_{j,SOV}^{CDS}\right) \right)$$

» An "**out-of-sample test**"

» Will the RSQs work for estimating the correlations between sovereigns and corporates?

» The question is important if we are optimizing a portfolio consisting of, say, both sovereign and corporate debt

RSQ Validation (2) : Out-of-Sample Comparison

$$corr\left(\sum_{j=1}^{N_{CORP}} corr\left(r_{i,SOV}^{CDS}, r_{j,CORP}^{CDS}\right), \sum_{j=1}^{N_{SOV}} corr\left(r_{i,SOV}^{CDS}, r_{j,SOV}^{CDS}\right)\right)$$

Region	Corr
Europe/M. East	0.82
Asia	0.91
Americas	0.81
Overall	0.84

Countries highly correlated with other countries are also highly correlated with corporate firms as well

$\Rightarrow$ Sovereign credit risk and corporate asset returns share a similar set of drivers

RSQ Validation (3) : Including Corporate CDS in OLS

Use CDS quotes directly in estimation

Estimate $$\log\left(\frac{corr(r_i^{CDS}, r_j^{CDS})}{corr(\phi_j, \phi_i)}\right) = \beta_i 1_i + \beta_j 1_j + \varepsilon_{i,j}$$

on the sample of safest 20 sovereigns and 400 corporates using both sovereign and corporate CDS spread information.

We find that the sovereign RSQs estimated using this method match the ones estimated using the original approach.

Conclusion

We devise and implement methodology that relies on CDS market data and GCorr factor structure to estimate the asset correlations between:

- Any two sovereign nations
- A sovereign nation and an entity from another asset class including:
  - 42,000 public firms
  - Private firms
  - CRE exposures
  - Retail exposures

These correlations can be imported in portfolio optimizer, risk system (such as MA RiskFrontier), or a valuation engine.

We validate the methodology using several methods.

About the Author:

Vojislav Sesum, Ph.D. works in the KMV Portfolio Research group at Moody's Analytics. He is responsible for various aspects of RiskFrontier, the Moody's Analytics portfolio solution used by the leading financial institutions. His current research interests include credit risk (especially concerning sovereign and municipal debt), asset pricing, and asset and liability management.

Prior to joining Moody's Analytics, Vojislav's experience includes working for Lehman Brothers in New York City. Dr. Sesum holds a Ph.D. in Finance from the Stanford Graduate School of Business, and a B.A. in Mathematics and Economics from Williams College.

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