GMO

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The Capacity of an Equity Strategy

Defining and Estimating the Capacity of a Quantitative Equity Strategy

Capacity is an intuitive but ill-defined concept linked to the diseconomies of scale in equity investing. The general idea is that as assets under management increase beyond the point where economies of scale are fully realized, liquidity and ownership constraints erode an equity investment strategy's ability to add value. This paper refines the definition of capacity and proposes two methodologies for estimating the capacity of a quantitatively managed equity strategy. A quantitative investment process lends itself to historical simulations where a controlled experiment can be run to determine the effect on performance of varying only one variable – assets under management (AUM). The sensitivity of capacity estimations to various assumptions, first and foremost about market impact costs, leads to a wide range of possible answers. Capacity, therefore, should not be thought of as a specific number, but as a variable of which we are trying to estimate the range of likely values.

Why Is Capacity Important

As emerging markets posted strong returns in recent years, a number of emerging markets equity managers have experienced tremendous interest in their asset class and large inflows of new money into their strategies. Given the relative illiquidity of the asset class, a number of firms have closed their emerging markets equity strategies in recent months.

The decision to close a strategy is a very important one for an investment management firm for it affects many aspects of its operations. From a business standpoint, closing a strategy limits the amount of revenues and therefore profits the firm can extract from it. From an investment management standpoint, closing a strategy preserves its ability to deliver value added (alpha).[1] From a marketing standpoint, closing a strategy improves the firm's brand by reinforcing the firm's integrity in the eyes of the marketplace and its commitment to putting its clients' interests ahead of its profit motives. Ideally, a strategy would be closed when it "reaches capacity," yet clearly defining and measuring capacity has been an elusive goal.

What Is Capacity?

The basic idea behind the concept of "capacity" is that the cost of implementing a strategy increases with assets under management (AUM) thereby eroding the strategy's ability to generate alpha.

Capacity is an intuitive concept but it is very hard to measure. The main problem is that the level of AUM is only one of many factors affecting the performance of a strategy. In the case of a strategy with a clear style bias (e.g., value, growth, momentum etc.), an increase in AUM might coincide with a period in which that style is in favour therefore leading to an overestimation of the true capacity of the strategy (and vice versa).

Characteristics imposed upon a strategy by increasing AUM might also affect performance and therefore the estimation of capacity. For example, as AUM increase, a strategy will gradually gravitate towards stocks and countries with greater liquidity. If countries and stocks with greater liquidity happen to be performing well as the strategy tilts towards them, one would tend to overestimate capacity.

[1] In this paper we will use the terms value added and alpha interchangeably.

Finally, if the issue of capacity is raised it is usually as a consequence of large flows into the strategy because of the popularity of its style or asset class. The very performance of the strategy might have been driven by large inflows into the strategy. This could lead to unrealistic assumptions as to the alpha of the strategy once interest in that style or asset class moderates or reverses.

Therefore, one reasonable way of studying capacity is in a simulated environment where the same strategy can be implemented under the same historical circumstances while changing only one control variable – AUM. Clearly, we need to be very cautious about interpreting the results obtained through historical simulations.

Definition of Capacity

In order to study capacity one first needs to define it. If one defines capacity as the amount of AUM that maximizes alpha, then an argument could be made that capacity is effectively zero, as Perold and Salomon [1991] pointed out: when a paper portfolio is implemented into an actual portfolio one sees a significant deterioration in alpha and the deterioration will gradually increase as AUM grow. One could argue that there are efficiencies of scale, especially for custody and ticket costs, but also in terms of negotiating better commission rates with brokers. Let's call "implementation capacity" the asset level at which those efficiencies are fully realized. The implementation capacity would have very little relationship to the alpha the strategy might be able to deliver.[2] We therefore need a different definition of capacity. We would like to propose three of them – threshold capacity, wealth maximizing capacity, and terminal capacity.

We will define **threshold capacity** the amount of AUM beyond which a strategy is no longer able to achieve its stated investment return objectives. For the purpose of this study let's assume the investment return objective is 5% annualized alpha net of transaction costs (but gross of management fees) over a business cycle.[3]

We will define **wealth maximizing capacity** the AUM that maximize the amount of wealth created (alpha times AUM), where alpha is net of transaction costs. Perold and Salomon [1991] argue that the right amount of AUM is the wealth maximizing capacity. Alpha is assumed to be declining more than linearly as AUM increase,[4] wealth created increases with AUM even as alpha declines, at least up to the wealth maximizing capacity.

We will define **terminal capacity** the level of AUM that reduces the net alpha to zero, in other words, the amount of AUM which causes transaction costs to be as high as the alpha the strategy can potentially deliver. In other words, at terminal capacity the alpha delivered after implementation costs is zero.

Note that, in the absence of performance fees, the aim of maximizing the investment management firm's revenues would lead to gathering assets in excess of even the terminal capacity (at least until its clients finally recognize the problem and make the decision to move their assets somewhere else). The financial incentives for the typical investment management firm are therefore to gather assets in excess of the wealth maximizing or threshold capacities.

We believe that the appropriate definition of capacity is the threshold capacity based on the strategy's investment objectives. An investment management firm should close a strategy when the AUM in a strategy reach the point beyond which the strategy's ability to deliver on its stated return objectives would be compromised.

Capacity is Not a Number

Increasing AUM causes two problems, one linked to the trading activity required to implement the strategy and one linked to the size of the positions being accumulated in the portfolio. These two problems will suggest two complementary methodologies for estimating the capacity of a strategy.

[2] For the purpose of this study we will assume that economies of case are fully realized by the time one reaches $1B in AUM.

[3] Through this study we will use a GMO emerging markets strategy as the reference point for all our examples. The strategy has a return objective of 5% (gross of management fees) per year over the benchmark over a business cycle with a tracking error of between 8% and 10%.

[4] Market impact costs are assumed to grow more than linearly with AUM and therefore alpha declines more than linearly with increases in AUM.

Even if we are using simulations to determine the capacity of an equity strategy according to the chosen methodology, we still have a number of choices to make: what time period are we going to use to estimate capacity? What assumptions will we make for various components of transaction costs? Are we going to scale down the simulation alpha by a certain factor to reflect the disappointing real-life performance of a typical back-tested simulation?

Given all these possible choices, capacity (threshold, wealth maximizing, or terminal) should not be viewed as an exact number but as a range of possible estimates.

Capacity Through the Lens of Transaction Costs

For simplicity, in this article we will use only one estimate of capacity for each of the two methodologies suggested above. The estimates will be built using the time period January 1993 to December 2004 and our best estimate of transaction costs.

We will initially tackle the first problem caused by increasing AUM and look at capacity through the lens of transaction costs. The capacity of a quantitative equity strategy is more easily studied since it lends itself to historical simulations.[5] We can run the same strategy through the exact same historical circumstances changing only one variable – AUM.

For this study we used the S&P/IFC Investable Composite as the benchmark and the universe. We ran a monthly optimization trading off alpha (based on the attractiveness of the country of membership and the value and momentum characteristics of a stock), the risk of deviating from the benchmark along a number of risk dimensions (countries, MSCI sectors, and size), and the cost of trading. We also had penalties for trading and holding large illiquid positions.[6]

We ran a number of simulations for the period January 1993 to December 2004, increasing AUM by $1B each time. As AUM increase, the optimizer gives up some alpha in order to reduce both transaction costs and the concentration of the portfolio, which results in a reduction in turnover.

The transaction costs assumptions we used were 20bps for commission costs and 30bps for each 10% of average daily volume traded – a conservative estimate based on our experience in investing in emerging markets. We then measured the "realized" annualized alpha of the simulated portfolios over the period under study for each level of AUM. Here are the results.

EXHIBIT 1
Simulation Results

AUM (in $B)	Alpha	Net Alpha	Transaction Costs	Net Value Added (in $M)	Turnover
1	11.10%	9.42%	1.68%	94.2	119.7%
2	10.71%	8.74%	1.97%	174.8	105.3%
3	10.40%	8.29%	2.11%	248.7	96.1%
4	10.17%	7.96%	2.21%	318.4	89.5%
5	9.96%	7.69%	2.27%	384.5	84.5%
6	9.76%	7.44%	2.32%	446.4	80.3%
7	9.58%	7.21%	2.37%	504.7	76.9%
8	9.40%	7.00%	2.40%	560.0	73.9%
9	9.23%	6.81%	2.42%	612.9	71.3%
10	9.08%	6.63%	2.45%	663.0	69.1%
11	8.93%	6.46%	2.47%	710.6	67.1%
12	8.79%	6.29%	2.50%	754.8	65.3%
13	8.65%	6.13%	2.52%	796.9	63.7%
14	8.53%	6.00%	2.53%	840.0	62.2%
15	8.41%	5.86%	2.55%	879.0	60.8%
16	8.29%	5.72%	2.57%	915.2	59.6%
17	8.17%	5.60%	2.57%	952.0	58.4%
18	8.06%	5.47%	2.59%	984.6	53.0%
19	7.96%	5.34%	2.62%	1014.6	56.3%
20	7.86%	5.24%	2.62%	1048.0	55.4%

Source: GMO

As AUM increase, the optimizer gives up some alpha in order to control transaction costs. Net value added is simply the product of net alpha and AUM – the total dollar amount extracted from the inefficiencies of the market by the strategy.

According to our definition, the threshold capacity is around $22B and the wealth maximizing capacity is around $50B (see Exhibit 2 on the following page). Since the portfolio construction process is aware of the increasing market impact costs as AUM increase, it will gradually move towards holding the benchmark –

[5] In a quantitative investment process investment decisions are to a great extent automated, therefore, one can know with greater certainty what *would* have occurred historically.

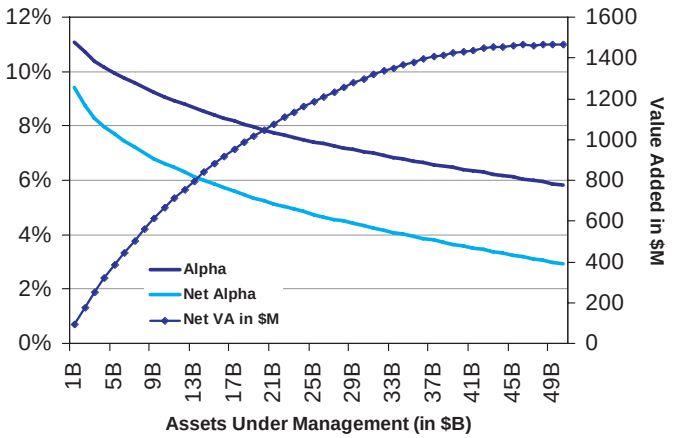
[6] We used a process and alphas very similar to those used to manage a GMO emerging market strategy.

effectively becoming an index fund. In other words, the terminal capacity is infinite.

Clearly, these estimates of capacity are based on the results of a series of historical simulations (a.k.a. backtests). Those of us who have extensive experience with simulated investment strategies know how dangerous it is to take their results at face value. Regardless of how realistic a transaction costs assumption one uses in a simulation may be, or how carefully one avoids survivorship biases, look-back biases, excessive data mining, and other backtesting foibles, real-life investment strategies always seem to fall short of the expectations created by the historical simulations that led to their development.[7]

EXHIBIT 2

The Effect of Increasing AUM on Alpha and Value Added



Source: GMO

To say that the threshold capacity is $22B, or that the wealth maximizing capacity is $50B, we must trust the accuracy of the historical simulations, a dangerous leap of faith indeed. So, rather than trusting the accuracy of the net alpha obtained by the historical simulations, we could look at the rate at which net alphas deteriorate with increases in AUM. One notices, for example, that going from AUM of $1B to $6B causes roughly a 20% deterioration in net alpha, and increasing AUM from $1B to $10B causes a 30% deterioration.

Let's assume that one managed to run a real-life strategy that exactly delivered on its investment return objective of 5% net alpha annualized since inception with a median of $1B under management. Then we could say that the threshold capacity of the strategy is $1B since we know from the historical simulation that any increase in AUM beyond $1B would have caused deterioration in performance and therefore would have prevented the strategy from delivering on its investment return objective. If, on the other hand, the same strategy had delivered, say 6.2% annualized since inception, then, based on the rate at which the net alpha deteriorated in our historical simulations, we could assume that the strategy has a threshold capacity of $6B. In other words, since the strategy delivered 20% more than promised with $1B under management, and knowing from the historical simulation that going from $1B to $6B costs a 20% reduction in net alpha, we could infer that the same deterioration in performance would have occurred in the real-life strategy due to a similar increase in AUM.

It is important to realize that this particular simulation and the capacity estimates derived from it are very much a function of the specific strategy tested. A strategy that relied more heavily on momentum for example, would lead, according to this method, to a lower estimate of capacity since the strategy would require a higher turnover to be implemented than a strategy with a greater emphasis on value.

Looking at Capacity Through the Lens of Breadth

The second problem with increasing AUM is the accumulation of large illiquid positions in the portfolio. This problem suggests one more way of looking at capacity and it has to do with possible constraints on the percentage ownership of stocks in the portfolio. The issue with capacity, in the final analysis, is being unable to buy enough of the stocks one likes, and therefore having to buy and own the second best ideas, and third best, and so on.

We define concentration as follows:

$$C = \sum_i W_{pi} * \left(\frac{W_{pi}}{W_{bi}} \right) \qquad \text{(I)}$$

Where W_{pi} is the weight of asset i in the managed portfolio, W_{bi} is the weight of asset i in the benchmark portfolio.

[7] A well-known quant joke goes as follows: "How do you insure the failure of an investment strategy? You backtest it."

Concentration is a measure of the ratio between the average weight of the assets held in the portfolio over their weight in the benchmark (in fact it is the weighted average of that ratio weighted by the portfolio weights). For example, a portfolio concentration of 5 means that the portfolio weight in an asset is on average roughly[8] five times its benchmark weight. This measure of concentration can be interpreted as the "distance" between the portfolio and the benchmark. If the portfolio has a concentration of one, then it is identical to the benchmark for each asset must be held at exactly the benchmark weight. The only way to take active positions is to hold some stocks at higher weight than their benchmark weight.

Therefore, to achieve a certain level of alpha the portfolio needs to have a certain level of concentration. Given a minimum required concentration, the amount of capital that can be deployed in the strategy is determined by the percentage of the free float of a stock one is willing to hold. For example, if one is required to have a minimum concentration of 3 and one is not willing to hold more than 1% of the free float of a stock, then the maximum amount of capital one can deploy in the strategy is 33 basis points of the total free float of the market.[9]

Next we need to study the relationship between alpha and concentration to determine the minimum level of concentration that would allow us to still deliver 5% alpha net of transaction costs.

Let's define breadth[10] as the inverse of the concentration.

$$B = \frac{1}{C} \qquad \text{(II)}$$

We ran a number of simulations using the same inputs as before but targeting a certain level of breadth.

EXHIBIT 3

Alpha, Transaction Costs and Turnover as a Function of Breadth

(Over the Period 1/93 to 12/04)

Breadth	Concentration	Gross Alpha	Alpha Net of Transaction Costs	Turnover
10%	10.0	15.9%	12.8%	130%
15%	6.7	14.7%	11.9%	129%
20%	5.0	13.5%	11.1%	126%
25%	4.0	12.3%	10.2%	121%
30%	3.3	11.1%	9.3%	116%
35%	2.9	10.0%	8.5%	109%
40%	2.5	9.0%	7.6%	102%
45%	2.2	7.9%	6.8%	94%
50%	2.0	6.8%	5.8%	85%
55%	1.8	5.4%	4.6%	74%
60%	1.7	3.5%	2.9%	57%

Source: GMO

If we trust the numbers from the backtest, it would seem that one would be able to deliver an alpha net of transaction costs of about 5% with a breadth in the neighbourhood of 50% to 55%.[11]

If that's the case, then we would be looking at the following capacity:

EXHIBIT 4A

Capacity as a Percentage of Market Capitalization

		Max Holding As Percentage of Float				
		1%	2%	3%	4%	5%
Portfolio Breadth	45%	0.45%	0.90%	1.35%	1.80%	2.25%
	50%	0.50%	1.00%	1.50%	2.00%	2.50%
	55%	0.55%	1.10%	1.65%	2.20%	2.75%
	60%	0.60%	1.20%	1.80%	2.40%	3.00%

Source: GMO

[8] If the benchmark and the portfolio are equal weighted then a portfolio with concentration C will own stocks at exactly C times their weight in the benchmark. If the benchmark or the portfolio is not equal weighted then the relationship is only approximately true.

[9] If on average we have to hold at least 3 times the benchmark weight in a stock, then we would have to hold all the stocks at 3 times their benchmark weight to maximize the capital we can deploy in the strategy. This means that we would hold in the portfolio stocks representing 33% of the market capitalization (we are assuming the portfolio has only long positions and is fully invested). Since we only want to hold 1% of the free float, the maximum amount of AUM in the strategy would be 33 basis points (33% * 1%) of the total market free float.

[10] The nice thing about breadth is that it goes from 0 to 1 (or 100%) while concentration goes from 1 to potentially a very large number (if the portfolio holds only one stock with a tiny weight in the benchmark).

[11] This, by the way, would mean that the strategy can hold stocks representing about half of the market capitalization. As a point of reference, the average breadth for the GMO emerging markets strategy has been around 17%, the actual alpha was 6.38% (annualized from inception 12/9/1993 to 12/31/2004), the actual tracking error was 7.92%, and turnover was around 70% per year.

EXHIBIT 4B

Capacity in $M (Based on S&P/IFCI Market Capitalization as of 12/31/2004)

		Max Holding As Percentage of Float				
		1%	2%	3%	4%	5%
Portfolio Breadth	45%	6,567	13,135	**19,702**	26,269	32,837
	50%	7,297	14,594	**21,891**	29,188	36,485
	55%	8,027	16,054	**24,080**	32,107	40,134
	60%	8,756	17,513	**26,269**	35,026	43,782

Source: GMO

Let's assume one is comfortable with holding on average 3% of the outstanding capitalization of a stock, then, if we trust the backtest, it would seem that the threshold capacity is around $23B. If we don't trust the numbers from the backtest (which would be a wise precaution), then we can use the "slippage" method introduced earlier.

Again, let's assume that the previously mentioned hypothetical strategy has delivered 6.2% alpha annualized since inception with a breadth 17%. We can then look at the breadth increase from 17%, which would cause a 20% decline in alpha. According to our analysis, moving from a breadth of 17% to a breadth of 30% would cause a 20% slippage in alpha. In other words, it would seem that if a strategy delivered 6.2% with a breadth of 17% it could have delivered 5% with a breadth of 30%. Therefore, the threshold capacity for the strategy would be as follows based on the maximum % of the free float of a stock one is willing to hold.

EXHIBIT 5A

Capacity as a Percentage of Market Capitalization Assuming Portfolio Breadth of 30%

Max Holding As Percentage of Float				
1%	2%	3%	4%	5%
0.3%	0.6%	0.9%	1.2%	1.5%

Source: GMO

EXHIBIT 5B

Capacity in $M (Based on S&P/IFCI Market Capitalization as of 12/31/2004) Assuming Portfolio Breadth of 30%

Max Holding As Percentage of Float				
1%	2%	3%	4%	5%
4,378	8,756	13,135	17,513	21,891

Source: GMO

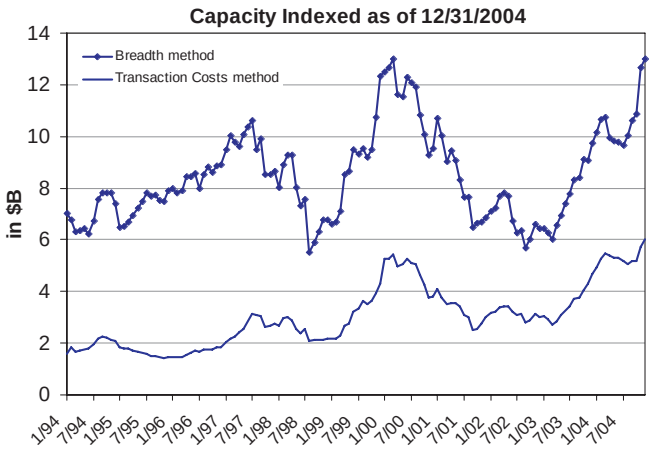
If one settles on a maximum ownership of 3% of the free float of a stock, then the threshold capacity for the strategy according to the liquidity method would be around $13B.

Looking at Capacity Over Time

An estimate of capacity is a function of current market conditions. If one has been managing an emerging market equity strategy for a few years, one would know that managing $2B during the Asia crisis of 1997 is a completely different experience than managing $2B now (at the end of 2004), when trading volumes are high and market cap has been increasing for the last couple of years. On a macro level, the two main drivers of capacity are total investable market capitalization and total investable trading volume. Our estimates of capacity have been obtained for the market conditions existing as of 12/31/2004. The capacity estimates derived looking at the impact of trading should therefore be indexed over time on the total investable trading volume. The capacity estimates derived looking at the impact of accumulating large positions should be indexed on the total investable market capitalization. Here is how it looks graphically.

EXHIBIT 6

Capacity Estimates over Time



Source: GMO

So we now have a dynamic range of possible estimates for capacity that provides a guide for determining when to close a strategy.

Clearly, measures to control the inflows into a strategy and a plan for closing it should be put in place as the level of AUM approaches the lower bound of the range of capacity estimates. The strategy should be hard closed (closed to all inflows) by the time it approaches the high end of the range.

A number of factors can modify the capacity constraints on a hard closed strategy. First, a hard closed strategy would be in redemption-only mode, so as clients rebalance down their allocation to emerging markets, the AUM would decline relative to the indexed capacity estimates. Note that if the market goes up 10%, chances are the AUM in the strategy would also appreciate by about 10% (the difference would be due to the strategy's alpha), so the position of the AUM relative to the indexed estimate of capacity based on the breadth method does not significantly change with market moves. But new capital issuance (for example the recent slew of IPOs in India and China or the relaxation of the Limited Investability Factor in Taiwan) would have the effect of increasing the capacity of a strategy according to both estimation methods. Also, any increase in the overall trading volume brought about by an increase in the average shares turnover would boost the transaction costs-based capacity estimate.

On the other hand, if and when a country graduates to developed status (Korea seems the most likely candidate in the next five years), it will remove a percentage of any emerging market benchmark liquidity and capitalization, effectively reducing the capacity of all emerging markets strategies (in the case of Korea the reduction in capacity would be somewhere between 20% and 30%).

Conclusions

We have attempted to define the concept of capacity more clearly and proposed two methodologies for arriving at an estimate of capacity of a quantitative equity strategy. Our work showed that there is no exact answer to the question, "What is the capacity of a strategy?" Yet, granted the uncertainty around such estimates, in order to preserve one's ability to deliver on one's stated investment objectives, it is important to have a framework for monitoring the capacity of a strategy and to guide decisions to close or re-open a strategy.

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Perold, Andre and Robert Salomon. "The Right Amount of Assets Under Management." *Financial Analyst Journal*, May/June 1991, pp. 31-39

[12] Mssrs. Bostock, Wilderman, Harris, Hancock, Inker, Berkley and Gray work at GMO.